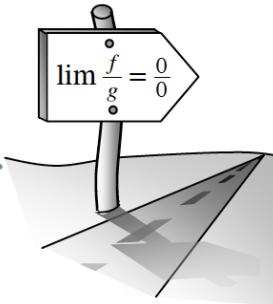


5.5.1 What are the limits?

Finding Limits of Indeterminate Forms



5-148. Without your calculator, find $\lim_{x \rightarrow 0} \frac{\sin x}{x}$. What happened? Now, graph the function $f(x) = \frac{\sin x}{x}$ and determine the limit.

5-149. Limits such as $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ are special because they cannot be determined simply by substitution.

Therefore, consider the values occurring in $f(x) = \frac{\sin x}{x}$ as $x \rightarrow 0$.

- Sketch $y = x$ and $y = \sin x$ in the window $x: [0, 0.5]$ by $y: [0, 0.5]$. What do you notice?
- How does the graph of $y = x$ and $\sin x$ help confirm the limit found in problem 5-148.

5-150. Use a similar approach to find $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$. Sketch $y = x$ and $y = 1 - \cos x$ on a new set of axes.

Write down some of your observations. Predict the limit and then confirm by graphing the function $y = \frac{1 - \cos x}{x}$ on your calculator.

5-151. Evaluate the following limits by considering the behaviors of the numerator and denominator as x approaches the limiting value. Justify your answers. A table or graph may help.

a. $\lim_{x \rightarrow \infty} \frac{x^2}{2^x}$

b. $\lim_{x \rightarrow 0^+} \frac{\log x}{x}$

5-152. As was proven in Chapter 2, for limits such as $\lim_{x \rightarrow 0} \frac{\sin x}{x}$, the limit can be found since it was easy to compare the behaviors of the functions $f(x) = \sin x$ and $f(x) = x$ near $x = 0$.

Other limits, called **indeterminate limits**, can be more difficult. They are in the form $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, 1^∞ , and 0^0 . They can all be reduced to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ using logarithms if necessary.

a. Explain why $\lim_{x \rightarrow 1} \frac{2x^3 + 7x^2 - 9}{x^3 + 3x^2 - 4}$ is an example of an indeterminate limit. Then find the limit and explain what method you used.

b. Let $f(x) = 2x^3 + 7x^2 - 9$ and let $g(x) = x^3 + 3x^2 - 4$. Graph $f(x)$ and $g(x)$ in the window $x: [0.75,$

$1.25]$ by $y: [-3, 3]$. Compare the graphs of $f(x)$ and $g(x)$ at $x = 1$. Compare their slopes. Do they have the same slope? Is one steeper than the other?

- c. Find $f'(x)$ and $g'(x)$ and compute $\lim_{x \rightarrow 1} f'(x)$ and $\lim_{x \rightarrow 1} g'(x)$.

5-153. Most of the difficult indeterminate forms are $\frac{f(x)}{g(x)}$ where $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ (or ∞). Look back at the previous problem to explain how looking at $f'(a)$ and $g'(a)$ might give you information about the value of the original quotient function.



5-154. Without using a graphing calculator, find the maximum and minimum values of the function $p(x) = (x^2 - 4)^2$ on the interval $[-1, 5]$. [Homework Help](#)

5-155. Use Newton's Method to calculate the x -intercept of the function $y = x - 2 + \sin x$, correct to 5 decimal places. [Homework Help](#)

5-156. Find the derivative of each of the following functions: [Homework Help](#)

a. $f(x) = x^2 \tan(x)$

b. $g(x) = \frac{1}{\sin x}$

c. $f(x) = (2x + 1)(3x - 1)^3$

d. $g(x) = \cos \sqrt{x+1}$

5-157. Construct a 50-term right-hand Riemann sum to estimate the area of the region bounded by the x -axis, the function $f(x) = \frac{1}{2^{x-1}}$, and the lines $x = 1$ and $x = 5$. Give the answer both numerically and using summation notation. [Homework Help](#)

5-158. Given $f(x) = x^4 - x^3$: [Homework Help](#)

- Find the equation of the tangent line at $(1, 0)$.
- Find the equation of the line normal to the tangent at $(1, 0)$.
- Find all points on $f(x)$ with the same slope as in part (b).

5-159. Consider the function defined as follows: $d(x) = \begin{cases} |4 - x^2| & \text{for } x < 0 \\ (x - 8)^{2/3} & \text{for } 0 \leq x \leq 16 \\ \frac{x-5}{3} & \text{for } x > 16 \end{cases}$ [Homework Help](#)

- a. Determine the differentiability of the function at $x = -2$ and $x = 8$. Explain why the function is or is not differentiable at each of these points.
- b. Determine all other points at which $d(x)$ might not be differentiable, and check the existence of the derivative at each point. For each point at which $d(x)$ is not differentiable, explain why not.

5-160. Bob's cat Alice does wind sprints up and down the hallway of his apartment. Suppose that t minutes into her wind sprints, Alice's velocity (in ft/min) is given by

the formula $v(t) = 12 \sin\left(\frac{\pi t}{4}\right)$. [Homework Help](#) 

- a. What is Alice's acceleration at $t = 6$ seconds?
- b. What is Alice's displacement between $t = 4$ and $t = 24$?
- c. What is the total distance traveled by Alice between $t = 2$ and $t = 8$?

