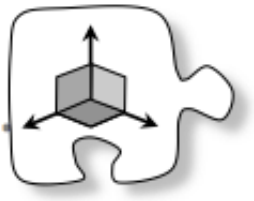
6.1.1 How can I plot points in three dimensions?



Creating a Three-Dimensional Model

In geometry, you worked with objects that existed in different dimensions. You considered lines and line segments, which have only one dimension: length. You also looked at flat shapes like circles, rectangles, and trapezoids that have two dimensions: length and width. Prisms, cones, and most objects that you encounter in the real world have volume, and therefore have three dimensions: length, width and height.

When you worked with graphs in Algebra 1, you represented points, the number line, and curves on a **two-dimensional** (flat) surface called the *xy*-plane. So far, you have only been able to represent relationships with at most two unknowns, usually the variables x and y. However, many problems, like some that you may have done in homework problems in Chapter 6, have more than two unknowns. Today, you and your team will build a model that will help you graph in three dimensions. As you work on this lesson, consider the following questions with your team:

How can we plot a point in three dimensions?

How can we write the coordinates of a point in three dimensions?

How can we show three dimensions on flat paper?

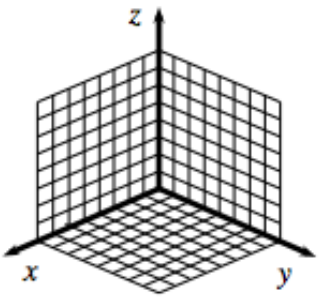
6-1. Consider when it is appropriate to graph a situation in one, two, and/or three dimensions. It may be helpful to think about your experience representing numbers and relationships on a number line or an *xy*-plane, and how you can adapt your knowledge to work in three dimensions. Discuss each question with your team before writing your response.

a. How can you represent the solution to $x = 5$ graphically? Can you think of more than one way?

b. How can you represent the solutions to $x + 2y = 5$ graphically?

c. How could you represent the solutions to $x + 2y + z = 5$? What would the solutions look like? Discuss these questions with your team and write down any ideas that you have.

6-2. To graph solutions to equations with three variables, you need to use a three-dimensional coordinate system. Obtain a Lesson 6.1.1A or 6.1.1B Resource Page from your teacher. Use scissors to cut out the region indicated on the page. Then fold along each of the axes and use tape to attach the dashed edge to the z-axis. Be sure that the grid ends up on the *inside* of your model (rather than the outside). The result should look similar to the diagram at right.



6-3. Place a penny (or other marker) on the bottom surface of your model at the point where $x = 4$ and $y = 2$. Now lift your marker straight up so that you are holding it 3 units above the bottom of the model.

a. With your team, find a way to write the coordinates of this point.

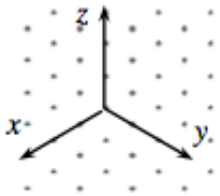
b. In your model, find the point where $x = 3$, $y = 4$, and $z = 2$. Use your team's method to write the coordinates for this point.

c. The model you have created is only a portion of the entire coordinate system used to represent three dimensions mathematically. How many of these models would you have to put together to create a model that represents the entire three-dimensional coordinate system? Think about the regions you would need to graph points like (5, −2, −7) or (−1, −2, −4).

6-4. Use cubes to build each shape described below inside your three-dimensional model. Make sure that one corner of each shape you build lies at the **origin** (at the point (0, 0, 0)).

a. Build a 2 x 2 x 2 cube. Use coordinates to name the vertex that is farthest from the origin.

b. Build a rectangular prism that is 2 units in length along the x-axis, 1 unit in length along the y-axis, and 3 units in length along the z-axis. Use coordinates to name the vertex that is farthest from the origin.



c. Draw and label a three-dimensional coordinate system on isometric dot paper like the one shown at right. Now add the prism from part (b) to the drawing. On your dot paper, label the coordinates of *all* of the vertices.

6-5. Build a rectangular prism that will have vertices in your model at (1, 0, 0), (0, 0, 4), and (0, 3, 0).

a. Find the coordinates of the other five vertices.

b. Move the rectangular prism so that three vertices are at (−1, 0, 0), (0, 0, 4), and (0, 3, 0). Now where are the other vertices?

c. Is it possible to build another rectangular prism that has the same coordinates for the vertex farthest from the origin as the prism in part (b)? Be sure to **justify** your conclusion.

6-6. On isometric dot paper, draw a three-dimensional coordinate system and plot the following points: (0, 1, −1), (1, 2, 0), and (2, 3, 1).

a. What do you notice about the three points?

b. With your team, find a strategy to make each point clearly different from the others. Be prepared to share your strategy with the class.

c. Identify the coordinates of two points that appear to be the same as (−2, 0, 0).

6-7. LEARNING LOG

In your Learning Log, show and explain how to graph points in three dimensions. Include clear pictures to illustrate your method. Title this entry "Plotting Points in xyz-Space" and label it with today's date.

6-8. Make a table like the one below. Choose points in each of the locations listed at the top of the table and write in the coordinates of the points you have chosen. Help (Html5)⇔Help (Java)

	Points on the x-axis	Points on the y-axis	Points on the z-axis	Points not on the x-, y-, or z-axes
1st point	(, ,)	(, ,)	(, ,)	(, ,)
2nd point	(, ,)	(, ,)	(, ,)	(, ,)
3rd point	(, ,)	(, ,)	(, ,)	(, ,)
4th point	(, ,)	(, ,)	(, ,)	(, ,)

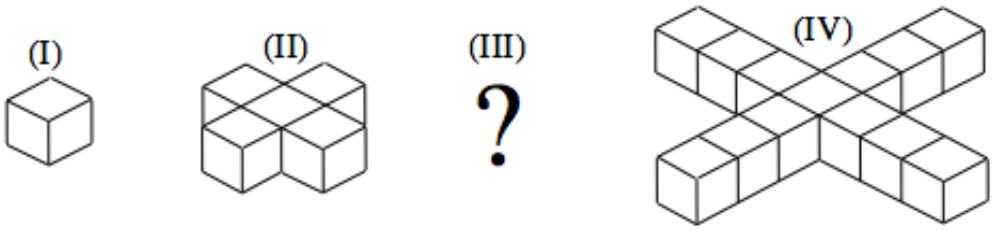
a. What do you notice about the coordinates of the points on the x-axis?

b. Make a conjecture about the coordinates of points that lie on any of the coordinate axes.

6-9. Solve the system of equations below. Help (Html5)⇔Help (Java)

$$3x + 8 = 2$$
$$7x + 3y = 1$$

6-10. Each cube below is 1 cm on a side. Help (Html5)⇔Help (Java)



a. Based on the pattern, find the volume of Figure III.

b. If the pattern continues, write an expression to represent the volume of figure N. What kind of sequence is this?

6-11. Solve each exponential equation for x. Help (Html5)⇔Help (Java)

a. $10^x = 16$

b. $10^x = 41$

c. $3^x = 729$

d. $10^x = 101$

6-12. Rewrite each expression below as an equivalent expression without negative exponents. Help (Html5)⇔Help (Java)

a. 5^{-2}

b. xy^{-2}

c. $(xy)^{-2}$

d. $a^3b^4a^{-4}b^6$

6-13. Multiply or divide and simplify each of the following expressions. Help (Html5)⇔Help (Java)

a. $\frac{3x}{x^2+2x+1} \div \frac{3}{x^2+2x+1}$

b. $\frac{3}{x-1} \cdot \frac{2}{x-2}$

6-14. Given the two points (−2, 0) and (0, 1), complete parts (a) through (c) below. Help (Html5)⇔Help (Java)

a. Find the slope of the line that passes through these two points.

b. Find the slope of the line perpendicular to the line that passes through these two points.

c. Describe the relationship between the slopes of perpendicular lines.

6-15. The cost of food has been increasing by 4% per year for many years. To find the cost of an item 15 years ago, Heather said, "*Take the current price and divide it by* $(1.04)^{15}$."

Her friend Elissa said, "*No, you should take the current price and multiply it by* $(0.96)^{15}$ *!*"

Explain who is correct and why. Help (Html5)⇔Help (Java)