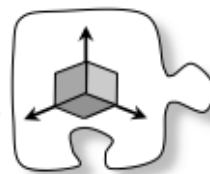


## 6.1.2 How can I graph an equation in three dimensions?

### Graphing Equations in Three Dimensions



In the past, you have used the two-dimensional Cartesian coordinate system ( $x$ - and  $y$ -axes) to graph equations involving two variables. In Lesson 6.1.1, you used a three-dimensional coordinate system to plot points. Today you will use the three-dimensional coordinate system to graph equations that have three variables. As you are working through the lesson, use the following questions to help focus your discussion:

How can we use what we know about graphing in two dimensions to help us graph in three dimensions?

What does a solution to a three-variable equation represent?

**6-16.** Consider the equation  $5x + 8y + 10z = 40$ .

- Discuss with your team what you think the shape of the graph would be. Explain how you decided.
- Is the point  $(4, 5, -2)$  a solution to the equation  $5x + 8y + 10z = 40$ ? **Justify** your answer.
- Your team will be given a list of points to test in the equation. Plot each point that makes the equation true on the three-dimensional graphing tool, [3-D Graph](#) (CPM).
- Now examine the solutions displayed on the graphing tool. With your team, discuss the questions below. Be ready to share your discoveries with the class.
  - Are there any points that you suspect are solutions, but do not have a point showing on the graph?
  - How many solutions do you think there are?
  - Are there any points showing that you think are not solutions? Explain.
  - What shape is formed by all of the solutions? That is, what is the shape of the graph of  $5x + 8y + 10z = 40$ ?



**6-17.** How can you graph an equation like  $12x + 4y + 5z = 60$  in three dimensions? To come up with a strategy to graph a three-variable equation, look at the strategies you can use to graph a two-variable equation in two dimensions. For example, consider  $5x + 8y = 40$ .

- What is the shape of the graph of  $5x + 8y = 40$ ? How can you tell?
- With your team, brainstorm all of the strategies you could use to graph  $5x + 8y = 40$ . Which strategy do you prefer? Why?

**6-18.** Now you will work with your team to graph  $12x + 4y + 5z = 60$ .

- What do you think it will look like?
- Which of the strategies you used to graph a two-variable equation in problem 6-17 can be used to graph this three-variable equation? Work with your team to find a strategy and then graph  $12x + 4y + 5z = 60$  on your isometric dot paper. Be prepared to share your strategy with the class.

**6-19.** Use your new strategy to graph each of the following equations in three dimensions.

a.  $13x + 4y + 5z = 260$

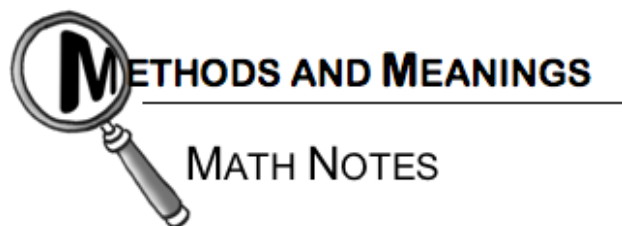
b.  $12x - 9y + 10z = 0$

**6-20.** Consider the graph of  $x = 4$  for each of the following problems.

a. Graph the solution to  $x = 4$  in one dimension (on a number line).

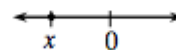
b. Graph the solutions to  $x = 4$  in two dimensions (on the  $xy$ -plane).

c. Graph the solutions to  $x = 4$  in three dimensions (in the  $xyz$ -space).

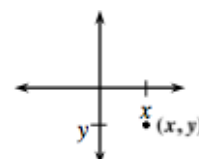


## Locating Points in Three Dimensions

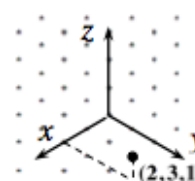
When locating a point on a *number line*, a single number,  $x$ , is used.



The location of a point in a *plane* is given by two numbers,  $(x, y)$ , called an ordered pair.



To locate a point in *space*, three numbers,  $(x, y, z)$ , are used, which are called an **ordered triple**. The point  $(2, 3, 1)$  is shown at right. The dotted lines help clarify which coordinate was graphed.



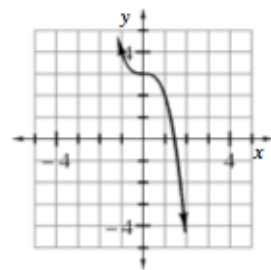
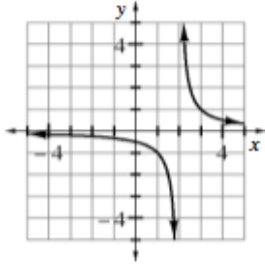
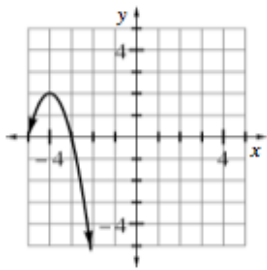
**6-21.** For each of the following equations, find every point where its *three-dimensional* graph intersects one of the coordinate axes. That is, find the  $x$ -,  $y$ - and  $z$ -*intercepts*. Express your answer in  $(x, y, z)$  form. [Help \(Html5\)](#)  $\leftrightarrow$  [Help \(Java\)](#)

- a.  $6y + 15z = 60$
- b.  $3x + 4y + 2z = 24$
- c.  $(x + 3)^2 + z^2 = 25$
- d.  $z = 6$

**6-22.** Answer each of the following questions. Illustrate your answers with a sketch. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

- a. What do you think the intersection of two planes looks like?
- b. What do you think it means for two planes to be parallel?
- c. What do you think it means for a line and a plane to be parallel?

**6-23.** Find an equation that will generate each graph. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

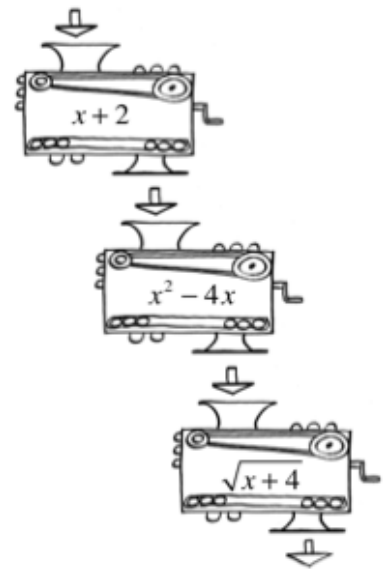


**6-24.** Is  $y = \frac{1}{x}$  the parent of  $y = \frac{1}{x^2 + 7}$ ? Explain your reasoning. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

**6-25.** Solve each equation below for  $x$ . [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

- a.  $2x + x = b$
- b.  $2ax + 3ax = b$
- c.  $x + ax = b$

**6-26.** Mark claims to have created a sequence of three function machines that always gives him the same number he started with. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)



- Test his machines. Do you think he is right?
- Be sure to test negative numbers. What happens for negative numbers?
- Mark wants to get his machines patented but has to prove that the set of machines will always do what he says it will, at least for positive numbers. Show Mark how to prove that his machines work for positive numbers by dropping in a variable (for example,  $n$ ) and writing out each step the machines must take.
- Why do the negative numbers come out positive?

**6-27.** Sketch the graph of  $y = \log_5(x - 2)$  and describe how the graph is transformed from the parent graph. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

**6-28.** The table at right shows the total population of Mexico for the given years. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

Year	Population (millions)
1900	13.6
1910	15.2
1920	14.4
1930	16.6
1940	19.8
1950	26.3
1960	35.0
1970	50.7
1980	69.7
1990	87.8
2000	100.3
2010	113.7

- What was the average rate of change for the population from 1900 to 1950?
- What was the average rate of change from 1960 to 2010?
- When was the population growth rate higher?

**6-29.** Given  $f(x) = -2x^2 - 4$  and  $g(x) = 5x + 3$ , calculate: [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

- $g(-2)$
- $f(-7)$
- $f(g(-2))$
- $f(g(1))$