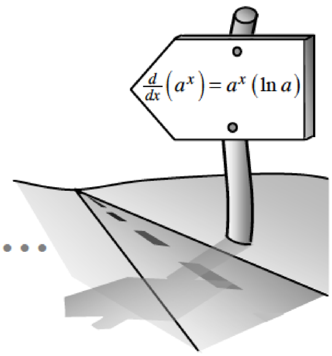
6.1.2 What is the difference between x^2, 2^x, and 3^x?

Derivatives of Exponential Functions



6-17. So far you have learned many derivative rules. However, there are still parent graphs you cannot yet differentiate.

a. Which parent equations can you not yet differentiate?

b. With your team, make a list of all the techniques we have used so far to find slope functions.

6-18. DERIVATIVES OF EXPONENTIAL FUNCTIONS, Part One
Find an untranslated, unstretched exponential function that is its own derivative. That is, given $y = a^x$, find a such that $\frac{d}{dx}(a^x) = a^x$.

6-19. NATURAL LOGARITHMS
Radians, which seem very unnatural at first, turn out to be the "natural" mathematical unit for measuring angles because $\frac{d}{dx}(\sin x) = \cos x$ only if x is measured in radians.

Similarly, $y = e^x$ turns out to be the "natural" exponential function because $\frac{d}{dx}(\sin x) = \cos x$ only if $a = e$. Therefore, it makes sense to call the *inverse* of $y = e^x$ the "natural" logarithmic function. This inverse function, $y = \log_e x$, has shorthand: $y = \ln x$(pronounced "el en of x") for " logarithmnatural" of x.

a. Use the natural logarithm function $y = \ln(x)$ to find an exact solution for $e^{2x-1} = 5$. Check your solution with your calculator.

b. Explain why $\ln e = 1$.

6-20. DERIVATIVES OF EXPONENTIAL FUNCTIONS, Part Two
Recall your approximation of $\frac{d}{dx}(2^x)$ from Lesson 6.1,1. Find an exact equation for $\frac{d}{dx}(2^x)$ using the fact that $\frac{d}{dx}(e^x) = e^x$.

If the exponential function can be expressed in terms of e, we can use our derivative rules to find the derivative of the function. We will start with $y = 2^x$:

a. Rewrite $y = 2^x$ in terms of e.

b. Use the Chain Rule to find the derivative of the expression found in part (a).

c. Combine your results form parts (a) and (b) to find $\frac{dy}{dx}$ using the original base.

d. Find a general expression for $\frac{d}{dx}(a^x)$.

6-21. Find the derivative of each function below.

a. $y = 5^x$

b. $y = 3 \cdot 7^x$

c. $y = \frac{1}{\ln 5} 5^x$

6-22. Find the derivative of each function below.

a. $3^{2x} + 3^{x-1} = f(x)$

b. $2^{x^3} \tan x = y$

c. $e^{\csc x} + 5^{2x} = g(x)$

Review & Preview

6-23. An electronics store purchases stereos from a supply company at a cost of \$63 each. They currently sell the stereos for \$99. At this price, the store is averaging 23 sales per week. Marketing studies show that for every \$2 decrease in price, the store will sell an additional 5 stereos. What price should the store use to maximize their profits? Homework Help ✎

6-24. First, decide if differentiating the following expressions requires the Product Rule, the Quotient Rule, the Chain Rule, or a combination of these rules. Then, find the following derivatives. Homework Help ✎

a. $\frac{d}{dt}\left[f(t^2)\right]$

b. $\frac{d}{dx}\left[x \cdot h(x)\right]$

c. $\frac{d}{dt}\left[t \cdot h(t^2)\right]$

6-25. Wei Kit is still looking for exponent shortcuts! Homework Help ✎

a. Since $2^{5+3} = 2^5 \cdot 2^3$, he thinks there is a way to simplify 2^{x+3} but needs your help. How can he rewrite 2^{x+3}?

b. Is 2^{x+3} proportional to 2^x ? Explain why or why not.

c. Use Wei Kit's method to rewrite 3^{x-2} and 5^{x+4}.

d. Does Wei Kit's method work on 3^{2x} ? Why or why not?

6-26. Examine the following derivatives. Consider the multiple tools available for finding derivatives and use the best strategy. After finding each derivative, write a short description of your method. Homework Help ✎

a. $\frac{d}{dx}\left[\sin x \cdot e^x\right]$

b. $\frac{d}{dz}\left[\frac{6z+1}{3z-2}\right]$

c. $\frac{d}{dt}\left[\tan t \cdot \cos t\right]$

6-27. Earlier, Jamal found the Riemann Sum $\sum_{i=0}^{9} \frac{1}{2} f\left(-3+\frac{1}{2}i\right)$, to estimate the area under $f(x) = 3x^2 - 2$ in the interval $[-3, 2]$ using 10 rectangles. Homework Help ✎

a. Use the summation feature of your calculator to find the approximate area using Jamal's Riemann sum.

b. Find the exact area using an integral expression.

6-28. Since $y = e^x$ and $y = \ln x$ are inverse functions, $e^{\ln x} = x$ and $\ln e^x = x$. Use these facts along with exponent and log laws to rewrite each expression. Homework Help ✎

a. $e^{2 \ln x}$

b. $e^{\ln \sqrt{x}}$

c. $\ln \sqrt{e^{5x}}$

d. $\ln(5e^x)$

6-29. Given: $f(x) = \begin{cases} 3x^{2/3} & \text{for } x \le 1 \\ a + bx^2 & \text{for } x > 1 \end{cases}$ where a and b are constants. Homework Help ✎

a. For what value(s) of x is the graph non-differentiable, regardless of the values of a and b? Explain what happens to $f(x)$ at these points.

b. Find values of a and b so the graph is both continuous and differentiable at $x = 1$.