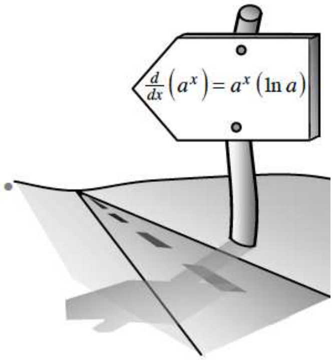
6.1.3 Can I combine derivative tools?



Derivatives Using Multiple Tools

Math Notes

Derivative of Exponential Functions

If $y = a^x$ where a is any (positive) base, then $\frac{dy}{dx} = (\ln a)a^x$. This means that the derivative of a^x is proportional to the original function where $\ln(a)$ is the proportionality constant. If $a = e,$ and because $\ln e = 1$, we have a very important special case:

$$\text{If } y = e^x \text{, then } \frac{dy}{dx} = (\ln e)e^x = e^x .$$

Thus e^x is its own derivative. Also, if $y = b \cdot a^x + k$ is a *general* exponential function, then $\frac{dy}{dx} = (b \ln a)(a^x)$.

The inverse of $y = e^x$ is known as the**natural logarithmic function**,abbreviated "ln x" (read as "el en of x").It is shorthand for "$\log_e x$." (Mathematicians tend to be extremely lazy!) Until you get used to this symbol, $\ln x$ can be replaced with $\log_e x$ when it appears in a problem.

6-30. Find each derivative below. Then check your solution with a graphing calculator.

a. $\frac{d}{dx}(e^x)$

b. $\frac{d}{dx}(7^x)$

c. $\frac{d}{dx}(3e^{x})\Big|_{x=1}$ (This means evaluate the derivative when $x = 1$.)

d. $\frac{d}{dx}(3\cdot 7^{x})\Big|_{x=1}$

6-31. Explain why $y = 3^{\cos x}$ is a composite function (that is, a function inside another function).

6-32. Differentiate the following expressions.

a. $\frac{d}{dx}(\sin(3x))$

b. $\frac{d}{dx}(5^{x})$

c. $\frac{d}{dx}(xe^{3x})$

d. $\frac{d}{dx}(5^{\cos x})$

e. $\frac{d}{dx}(\sin(3^{x}))$

f. $\frac{d}{dx}(x^{3}e^{x^{2}-5})$

g. $\frac{d}{dx}(2^{\cot x})$

h. $\frac{d}{dx}(e^{\tan x})$

i. $\frac{d}{dx}(6\cdot 2^{\cot x})$

j. $\frac{d}{dx}(10^{\sec x+\csc x})$

6-33. A collector's edition silver dollar is increasing in value at the rate of 10% per year. When it was made, its value was exactly \$1. How fast (in cents/year) is its value increasing when it is 10 years old?

Review & Preview

6-34. According to the State Department of Finance, California's population was 33.218 million people at the beginning of 1998, 33.765 million at the beginning of 1999, and 34.336 million at the beginning of 2000. Homework Help

a. Find the percent increase from 1998 to 1999 and the percent increase from 1999 to 2000. Does this suggest exponential population growth?

b. Assuming exponential population growth, find a model that approximately fits this data.

c. Use your model to predict California's population in 2020 assuming your growth model remains valid.

d. The increase in population is proportional to the current population. In other words, if the population is growing 3% each year, then if the population is 1,000 people, the increase is 30 people. If the population is 5,000 people, the increase is 150 people. Approximately how much should California's population have increased in the year 2000?

e. Explain why the increase is proportional to the population. What is the constant of proportionality?

6-35. Examine the integrals below. Consider the multiple tools available for integrating and use the best strategy. After evaluating each integral, write a short description of your method. Homework Help

a. $\int (6^x - 3\sec x \tan x)dx$

b. $\pi\int_0^2 ((2x)^2 - (x^2)^2)\,dx$

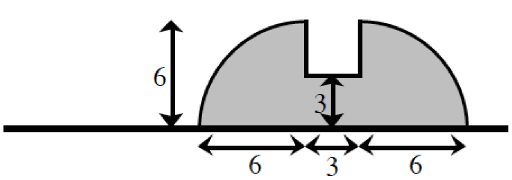
c. $\int_1^4 \frac{3x^2+13x-10}{x+5}\,dx$

d. $\int 6m^{-4/3}\,dm$

6-36. Sketch one continuous function that satisfies all of the following conditions. Read carefully - some are limits of the *derivative,* not the function. Where is $f(x)$ non-differentiable? Homework Help

- $\lim_{x \to -\infty} f(x) = \infty$
- $\lim_{x \to 0^-} f'(x) = -1$
- $\lim_{x \to 0^+} f'(x) = 1$
- $\lim_{x \to \infty} f(x) = 5$

6-37. Another flag is shown below. Homework Help



a. Imagine rotating the flag about its pole and describe the resulting three-dimensional figure. Draw a picture of this figure on your paper.

b. Find the volume of the rotated flag.

6-38. Differentiate the following functions with respect to the given independent variable. Homework Help ✎

a. $y = 2^{\log_2 x}$

b. $y = \tan 10^x$

c. $y = \cos t \tan t$

d. $y = \tan(e^x)$

e. $y = 2^{\cos(w^3)}$

6-39. VERTICAL MOTION

The acceleration due to gravity is a constant near the surface of the Earth. Its accepted values are −32.2 ft/sec^2 or −9.8 m/sec^2 . These values are all the information needed to derive the equations $v(t)$ and $a(t)$ for vertical motion for any dropped or thrown object. Homework Help ✎

a. Describe how can you find a velocity function when given the acceleration function.

b. Find $v(t)$ if $a(t) = -32$ ft/sec^2 and if the initial velocity of an object, v_0 , is 120 feet per second.

c. Find its height $s(t)$ if the starting position of the object, x_0 , is 100 feet above ground.

d. Find the velocity of the object when it hits the ground.

e. Find the maximum height attained by the object.

f. At what time is the object's speed greatest?

6-40. If $f(x)$ is a continuous function, $f'(2) = -1$, and $f'(3) = 1$, does there have to be an x-value a such that $2 < a < 3$ and $f'(a) = 0$? Draw a sketch to support your explanation. Homework Help ✎