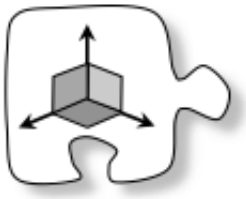
# 6.1.4 What is a solution in three dimensions?



## Solving Systems of Three Equations with Three Unknowns

Today you will extend what you know about systems of equations to examine how to solve systems of equations with three variables. As you work with your team, look for connections to previous work. The focus questions below can help generate mathematical discussion.

What does a solution to a system in three variables mean?

What strategies can we use?

What does the intersection look like?

**6-44.** Review the strategies for solving systems that you already know as you solve the following two-variable system of equations. Use any method. Do not hesitate to change strategies if your first strategy seems cumbersome. If there is no solution, explain what that indicates about the graph of this system. Leave your solution in $(x,y)$ form.

$$12x - 2y = 16$$
$$30x + 2y = 68$$

**6-45.** Solve the following three-variable system of equations by graphing it with your graphing tool 3-D Graph (CPM) or on isometric dot paper. Give your solution in $(x, y, z)$ form. Then test your solution in the equations and describe your results.

$$2x + 3y + 3z = 6$$
$$6x - 3y + 4z = 12$$
$$2x - 3y + 2z = 6$$

**6-46.** FINDING AN EASIER WAY

As you saw in problem 6-45, using a graph to solve a system of three equations with three variables can lead to inconclusive results. What other strategies should be considered? Discuss this with your team and be prepared to share your ideas with the class.

**6-47.** Looking at the equations in problem 6-45, Elissa wanted to see if she could apply some of her solving techniques from two-variable equations to this three-variable system.

a. Elissa noticed that the first two equations could be combined to form the new equation $8x + 7z = 18$. How did she accomplish this? Explain.

b. Now that Elissa has an equation with only $x$ and $z$, she needs to find another equation with only $x$ and $z$ to be able to solve the system. Choose a different pair of equations to combine and find a way to eliminate $y$ so that the new equation only has $x$ and $z$. Then solve the system to find $x$ and $z$.

c. For which variable do you still need to solve? Work with your team to solve for this variable. Then write

the solution as a point in $(x, y, z)$ form.

d. Is your solution reasonable? Does it make sense? Does it agree with your graph?

**6-48.** Practice using your algebraic strategies by solving the systems below, if possible. If there is no solution or if the solution is different than you expected, use the graphing tool to help you figure out why.

a. $$\begin{aligned} x+y+3z&=3\\ 2x+y+6z&=2\\ 2x-y+3z&=-7 \end{aligned}$$

b. $$\begin{aligned} 20x+12y+15z&=60\\ 20x+12y+15z&=120\\ 10x+20z&=30 \end{aligned}$$

c. $$\begin{aligned} 5x-4y-6z&=-19\\ -2x+2y+z&=5\\ 3x-6y-5z&=-16 \end{aligned}$$

d. $$\begin{aligned} 6x+4y+z&=12\\ 6x+4y+2z&=12\\ 6x+4y+3z&=12 \end{aligned}$$

**6-49.** Today you developed a way to solve a system of three equations with three variables. But what do the solutions of a system like those provided in problem 6-48 represent? Consider this as you answer the questions below with your graphing tool.

a. One of the systems in problem 6-48 had no solution. Graph this system with your graphing tool. Describe how the planes are positioned and why there is no common point on all three planes.

b. In what other ways could three planes be positioned so that there is no solution? Use paper or cardboard to help you communicate your ideas with others.

c. Graph the system in part (d) of problem 6-48 with your graphing tool and examine the result. How can you describe the intersection of these planes?

**6-50.** LEARNING LOG

In your Learning Log, describe your algebraic strategy to solve a system of three equations with three variables. Give enough details to allow you to repeat the process when you refer to it later. Title this entry "Systems of Three Equations with Three Variables" and include today's date.

METHODS AND MEANINGS

MATH NOTES

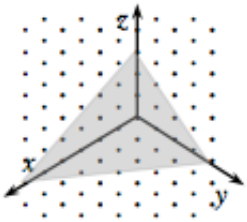
## Graphing Planes in Three Dimensions

To graph a plane, it is easiest to use the intercepts to draw the trace lines (the intersections of the plane with the *xy-, xz-,* and *yz*-planes) that will represent the plane.

To find the intercepts, let two of the variables equal zero. Then solve to find the intercept corresponding to the remaining variable.

For example, for $2x + 3y + 4z = 12$, the $x$-intercept is found by letting $y$ and $z$ equal zero, which gives $2x = 12$. Therefore the $x$-intercept is (6, 0, 0). Similarly, the $y$-intercept is (0, 4, 0), and the $z$-intercept is (0, 0, 3).

Drawing the line between two intercepts gives the trace line for the plane. For example, connecting the $x$- and $y$- intercepts, you would get the equation $2x + 3y = 12$, which is the trace line in the $xy$-plane when $z = 0$ in the equation $2x + 3y + 4z = 12$. Connecting the $x$- and $z$-intercepts gives the trace line in the $xz$-plane, etc.




Review & Preview

**6-51.** Use the algebraic strategies you developed in today's lesson to solve the system of equations below. Be sure to check your solution. Help (Html5)⇔Help (Java)

$$2x + y - 3z = -12$$
$$5x - y + z = 11$$
$$x + 3y - 2z = -13$$

**6-52.** Suppose that a two-bedroom house in Nashville is worth $110,000 and appreciates at a rate of 2.5% each year. Help (Html5)⇔Help (Java)

a. How much will it be worth in 10 years?

b. When will it be worth $200,000?

c. In Homewood, houses are depreciating at a rate of 5% each year. If a house is worth $182,500 now, how much will it be worth two years from now?

**6-53.** Solve $\sqrt{5x-1}=\sqrt{6+4x}$ and check your solution. Help (Html5)⇔Help (Java)

**6-54.** If two quantities are equal, are their logarithms also equal? Consider the questions below. Help (Html5)⇔Help (Java)

a. Is it true that $4^2$ is equal to $2^4$? Is this a special case, or is $a^b$ equal to $b^a$ for any values of $a$ and $b$?

b. Is $\log 4^2$ equal to $\log 2^4$? How can you be sure?

c. Are the equations $x = 5$ and $\log x = \log 5$ equivalent? **Justify** your answer.

d. Is the equation $\log 7 = \log x^2$ equivalent to the equation $7 = x^2$? How can you be sure?

**6-55.** Use the ideas from problem 6-54 to help you solve the following equations. Help (Html5)⇔Help (Java)

a. $\log 10 = \log(2x - 3)$

b. $\log 25 = \log(4x^2 - 5x - 50)$

**6-56.** Find an equation for each of the lines described below. Help (Html5)⇔Help (Java)

a. The line with slope $\frac{1}{3}$ that goes through the point (0, 5).

b. The line parallel to $y = 2x - 5$ that goes through the point (1, 7).

c. The line perpendicular to $y = 2x - 5$ that goes through the point (1, 7).

d. The line that goes through the point (0, 0) so that the tangent of the angle it makes with the $x$-axis is 2.

**6-57.** Solve each equation below for $y$ so that it can be entered in the graphing calculator. Help (Html5)⇔Help (Java)

a. $x^2 = x(2x - 4) + y$

b. $x = 3 + (y - 5)^2$

**6-58.** Sketch the graph of each equation or inequality below. Help (Html5)⇔Help (Java)

a. $(x - 2)^2 + (y + 3)^2 = 9$

b. $(x - 2)^2 + (y + 3)^2 \geq 9$

**6-59.** You are standing 60 feet away from a five-story building in Los Angeles, looking up at its rooftop. In the distance you can see the billboard on top of your hotel, but the building is completely obscured by the one in front of you. If your hotel is 32 stories tall and the average story is 10 feet high, how far away from your hotel are you? Help (Html5)⇔Help (Java)