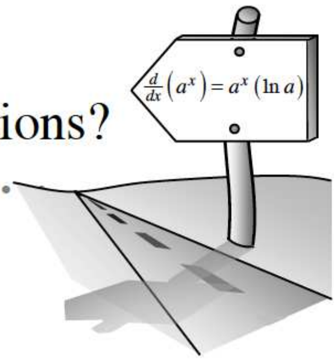
6.1.4 How do I integrate exponential functions?



Integrals of Exponential Functions

6-41. INTEGRALS OF EXPONENTIAL FUNCTIONS

Find each of these indefinite integrals. Test your answer by differentiating. Do not forget the constant of integration!

a. $\int e^x dx$

b. $\int 8e^x dx$

c. $\int (\ln 2)2^x dx$

d. $\int 2^x dx$

e. $\int 5 \cdot 2^x dx$

f. $\int 9^t dt$

6-42. Generalize your work from problem 6-41. That is, find $\int a^x \, dx$ for any positive constant a.

6-43. EXTENDED CHAIN RULE

a. Find $f'(x)$ if $f(x) = \tan(3x^2 - 1)$.

b. Use the result from part (a) to find $g'(x)$ if $g(x) = \sqrt{\tan(3x^2 - 1)}$.

c. The function $g(x)$ in part (b) is a composite of *three* distinct functions. Generalize your process. That is, if $f(x) = g(h(k(x)))$, find an expression for $f'(x)$ in terms of the other functions and their derivatives.

d. How can the Chain Rule be applied to compositions of more than three functions? Using complete sentences, describe the general process for differentiating compositions.

6-44. EXTENDED PRODUCT RULE

Likewise, study the use of the Product Rule on functions with more than two factors. Start with a simpler function, such as $f(x) = x \cdot 2^x$, to refresh your understanding of the Product Rule. Then, examine more complicated products with 3 or 4 factors. Finally, generalize the Product Rule to find $p'(x)$ if $p(x) = f(x) \cdot g(x) \cdot h(x)$.

6-45. Examine the derivatives below. Consider the multiple tools available for finding derivatives and use the best strategy. After finding each derivative, write a short description of your method.

a. $\frac{d}{dx}(7^{3\cos(2x)})$

b. $\frac{d}{dx}(2\sin e^{5x})$

c. $\frac{d}{dt}(t^2 \sin t \sec t)$

d. $\frac{d}{dt}(\tan(e^{t^2+1}))$

e. $\frac{d}{dx}(2e^{\sin(5x)})$

f. $\frac{d}{dx}(e^x \cot x)$

g. $\frac{d}{dt}\left[(4\sin^2 t)^{-2}\right]$

h. $\frac{d}{dx}\left[\sqrt[3]{x}\csc(2x)\right]$

Review & Preview

6-46. During a race Victor traveled with a velocity (in feet per second) of $v(t) = 2^t$ where t is measured in seconds. He won the race in t =6.136 seconds. Approximately how long was the race? It may help to graph first. Homework Help ✎

6-47. Sketch a graph of $x^2 + y^2 = 4$. Predict the slope of the tangents at $x = 0$. Then, confirm this algebraically by differentiating. (Hint: Solve for y first.) Homework Help ✎

6-48. On the moon, the acceleration due to gravity is about -5 ft/sec^2. An astronaut jumps into the air with an initial upward velocity of 12 ft/sec. Homework Help ✎

a. How long is the astronaut off the ground?

b. How far off the ground does the astronaut rise?

6-49. Differentiate the following expressions. Homework Help ✎

a. $\frac{d}{dz}\left[2z^2 \sec(z^3 - 2)\right]$

b. $\frac{d}{dt}\left[\sin(\tan^2(3t - 5))\right]$

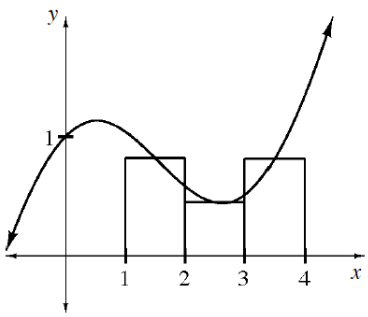
c. $\frac{d}{dx}\left[x \cdot 2^x \cos x\right]$

6-50. Find the first and second derivative for each function. Use these to test for maxima and minima in the given interval. Remember to check the endpoints. Homework Help ✎

a. $y = 2xe^x$ on $(-\infty, \infty)$

b. $y = \frac{x}{x^2+1}$ on $[-2, 2]$

6-51. Midpoint rectangles have their height defined at the midpoint of the interval. For the function $f(x) = \frac{1}{2}x + \cos x$ graphed below, you previously found the heights to be $f(1.5) \approx 0.821$, $f(2.5) \approx 0.449$, $f(3.5) \approx 0.814$. Homework Help ✎



a. Approximate $A(f,\ 1 \le x \le 4)$ using midpoint rectangles.

b. Find the exact area and compare it with your approximation.

6-52. Without a calculator, solve for x: $\left(\frac{1}{1+\left(\frac{5}{x}\right)^2}\right)\left(\frac{-5}{x^2}\right) - \left(\frac{1}{1+\left(\frac{1}{x}\right)^2}\right)\left(\frac{-1}{x^2}\right) = 0$. Homework Help ✎

MATH NOTES

Integrals of Exponential Functions

$$\int a^x\,dx = \frac{1}{\ln a}\cdot a^x + C$$

If $a = e$, and because In $e = 1$, we have a very important special case:

$$\int e^x\,dx = e^x + C$$

Thus, the integral (or antiderivative) ofe^x differs from the original function by at most a constant.

6-53. Evaluate. Homework Help ✎

a. $\lim\limits_{n\to 0} \dfrac{\int_2^{2+n}\left(\sqrt{x^3+1}\right)dx}{n}$

b. $\lim\limits_{x\to \pi} \dfrac{\int_\pi^x \cos t^2\,dt}{x-\pi}$

c. $\lim\limits_{x\to -\infty} \dfrac{x^2}{e^{-x}}$

d. $\lim\limits_{x\to 0} \dfrac{\sin(x^2)}{x\tan(x)}$ (Hint: Use L'Hospital's Rule)

6-54. Examine the integrals below. Consider the multiple tools available for integrating and use the best strategy. After evaluating each integral, write a short description of your method. Homework Help ✎

a. $\int (\ln 3)3^x\,dx$

b. $2\pi\int_0^2 x(2x+5)dx$

c. $\int 4e^{\ln x}\,dx$

d. $\int 2^{m+2}\,dm$