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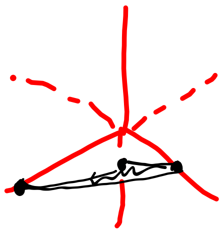
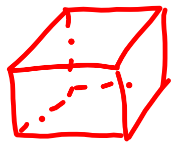
| Cornell Notes: | Name: |
| | Assignment: 6.1.4 Solving Systems of Three Equations with Three Unknowns |
| | Class: Algebra II |
| Essential Question: What is a solution in three dimensions? | |
| | Notes |
| WARM-UP | 1.) Plane ✓<br>2.) yes. ✓ $2(6)+3(11)-5(3)=30$ |

$12+33-15 = 30$

$30=30$





Today you will extend what you know about systems of equations to examine how to solve systems of equations with three variables. As you work with your team, look for connections to previous work. The focus questions below can help generate mathematical discussion.

What does a solution to a system in three variables mean?

What strategies can we use?

What does the intersection look like?

**6-44**

Review the strategies for solving systems that you already know as you solve the following two-variable system of equations. Use any method. Do not hesitate to change strategies if your first strategy seems cumbersome. If there is no solution, explain what that indicates about the graph of this system. Leave your solution in $(x, y)$ form.

$y = mx + b$

x-int, y-int

What are three ways to solve a system of equations?

Algebraic / Graphing

Substitution, elimination, graphing

$12x - 2y = 16$
$30x + 2y = 68$

$\left(\frac{16}{12}, 0\right), (0, -8)$

$\left(\frac{68}{30}, 0\right), (0, 34)$

**Substitution**

$12x - 2y = 16$
$-2y = -12x + 16$
$y = 6x - 8$

$30x + 2(6x - 8) = 68$
$30x + 12x - 16 = 68$
$42x = 84$
$x = 2$

$12(2) - 2y = 16$
$-2y = -8$
$y = 4$

$\boxed{(2, 4)}$

**Elimination**

$12x - 2y = 16$
$30x + 2y = 68$
$42x = 84$
$x = 2$

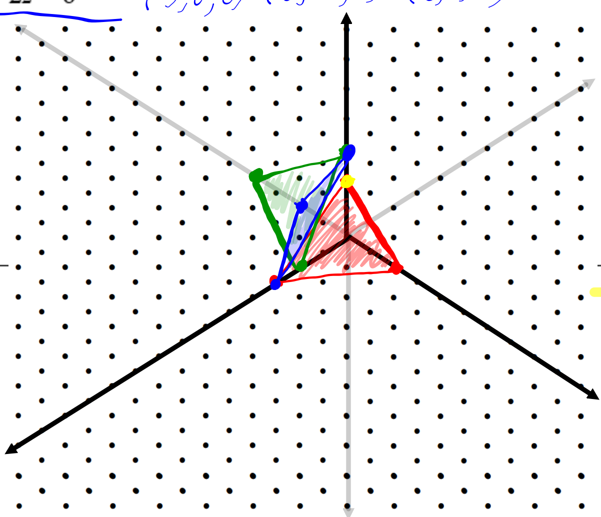
$12(2) - 2y = 16$
$y = 4$

$\boxed{(2, 4)}$

**6-45**

Solve the following three-variable system of equations by graphing it with your graphing tool 3-D Graph (CPM) or on isometric dot paper. Give your solution in $(x, y, z)$ form. Then test your solution in the equations and describe your results.

$2x + 3y + 3z = 6$ — $(3,0,0)$ $(0,2,0)$ $(0,0,2)$
$6x - 3y + 4z = 12$ — $(2,0,0)$ $(0,-4,0)$ $(0,0,3)$
$2x - 3y + 2z = 6$ — $(3,0,0)$ $(0,-2,0)$ $(0,0,3)$



$(0, 0, 2)$
$(2, -0.5, 0)$
$(1, 0, 3)$
$(0.5, -2, 1.8)$
$(1, 0, 2)$
$(1, -0.5, 1)$
$(1.5, 0, 3)$
$(1, -1, 1)$

6-46
FINDING AN EASIER WAY

As you saw in problem 6-45, using a graph to solve a system of three equations with three variables can lead to inconclusive results. What other strategies should be considered? Discuss this with your team and be prepared to share your ideas with the class.

Elimination and substitution

6-47

Looking at the equations in problem 6-45, Elissa wanted to see if she could apply some of her solving techniques from two-variable equations to this three-variable system.

1.) get two equations with only two variable use elim to do this

2.)Solve the new system of two equations with two variable (AGAIN elimination is best)

3.) At this point you know the value of two of the variables, now substitute to any of the originals and solve for the last variable

a. Elissa noticed that the first two equations could be combined to form the new equation $8x + 7z = 18$. How did she accomplish this? Explain.

Elimination

$2x + 3y + 3z = 6$
$6x - 3y + 4z = 12$
$8x + 7z = 18$

b. Now that Elissa has an equation with only $x$ and $z$, she needs to find another equation with only $x$ and $z$ to be able to solve the system. Choose a different pair of equations to combine and find a way to eliminate $y$ so that the new equation only has $x$ and $z$. Then solve the system to find $x$ and $z$.

$2x + 3y + 3z = 6$
$2x - 3y + 2z = 6$
$4x + 5z = 12$ $\times -2$
$8x + 7z = 18$ $\times 1$

$-8x - 10z = -24$
$8x + 7z = 18$
$-3z = 6$
$z = 2$

$4x + 5(2) = 12$
$4x = 2$
$x = \frac{2}{4}$
$x = \frac{1}{2}$

c. For which variable do you still need to solve? Work with your team to solve for this variable. Then write the solution as a point in $(x, y, z)$ form.

$y$

$2(\frac{1}{2}) + 3y + 3(2) = 6$
$1 + 3y + 6 = 6$
$3y = -1$
$y = -\frac{1}{3}$

$(\frac{1}{2}, -\frac{1}{3}, 2)$

d. Is your solution reasonable? Does it make sense? Does it agree with your graph?

**6-48** Practice using your algebraic strategies by solving the systems below, if possible. If there is no solution or if the solution is different than you expected, use the graphing tool to help you figure out why.

① $x+y+3z=3$
② $2x+y+6z=2$
③ $2x-y+3z=-7$

②+③
$2x+y+6z=2$
$2x-y+3z=-7$
Ⓐ $4x+9z=-5$

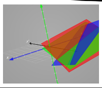
①+③
$x+y+3z=3$
$2x-y+3z=-7$
Ⓑ $3x+6z=-4$

Ⓐ+Ⓑ
$4x+9z=-5 \xrightarrow{\times -3} -12x-27z=15$
$3x+6z=-4 \xrightarrow{\times 4} 12x+24z=16$
$-3z=-1$
$z=\frac{1}{3}$

$3x+6(\frac{1}{3})=-4$
$3x+2=-4$
$3x=-6$
$x=-2$

$x+y+3z=3$
$(-2)+y+3(\frac{1}{3})=3$
$-2+y+1=3$
$y=4$

$\boxed{(-2,4,\frac{1}{3})}$

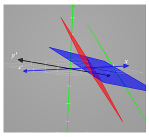


① $20x+12y+15z=60$
② $20x+12y+15z=120$
③ $10x+20z=30$

LOOK what happened here... we tried to get another equation with only x and z (like the third) but when we eliminated y we got this 0 = -60 which is not true... this means that there is no solution...

① + −1·②
$20x+12y+15z=60$
$-20x-12y-15z=-120$
$0=-60$ ???

No Solution



① $5x-4y-6z=-19$
② $-2x+2y+z=5$
③ $3x-6y-5z=-16$
notice: eliminate one variable

①+2·②
$5x-4y-6z=-19$
$-4x+4y+2z=10$
Ⓐ $x-4z=-9$

3·② + ③ Notice: use two equations (not the same ones), \to eliminate the same variable... AGAIN
$-6x+6y+3z=15$
$3x-6y-5z=-16$
Ⓑ $-3x-2z=-1$

$\boxed{(1,\frac{1}{2},2)}$

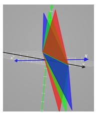
3·Ⓐ + Ⓑ
$3x-12z=-27$
$-3x-2z=-1$
$-14z=-28$
$z=2$

$x-4(2)=-9$
$x=-1$

$3x-6y-5z=-16$
$3(1)-6y-5(2)=-16$
$-6y-13=-16$
$-6y=-3$
$y=\frac{1}{2}$

$6x+4y+z=12$
$6x+4y+2z=12 \xrightarrow{\times -1} -6x-4y-2z=12$
$6x+4y+3z=12 \xrightarrow{\times 1} 6x+4y+3z=12$
$z=0$

Notice!!! plugging in z=0 to all of these equations gives........ 6x+4y=12... the answer then is the whole line in three dimensions 6x+4y=12



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| 6-49 | Today you developed a way to solve a system of three equations with three variables. But what do the solutions of a system like those provided in problem 6-48 represent? Consider this as you answer the questions below with your graphing tool.<br><br>a. One of the systems in problem 6-48 had no solution. Graph this system with your graphing tool. Describe how the planes are positioned and why there is no common point on all three planes.<br><br>b. In what other ways could three planes be positioned so that there is no solution? Use paper or cardboard to help you communicate your ideas with others.<br><br>c. Graph the system in part (d) of problem 6-48 with your graphing tool and examine the result. How can you describe the intersection of these planes? |