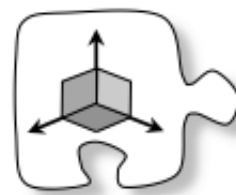


6.1.5 How can I apply systems of equations?



Using Systems of Three Equations for Curve Fitting

In this lesson you will work with your team to find the equation of a quadratic function that passes through three specific points. You will be challenged to extend what you know about writing and solving a system of equations in two variables to solving a system of equations in three variables.

6-60. In your work with parabolas, you have developed two forms for the general equation of a quadratic function: $y = ax^2 + bx + c$ and $y = a(x - h)^2 + k$. What information does each equation give you about the graph of a parabola? Be as detailed in your explanation as possible. When is each form most useful?

6-61. Suppose the graph of a quadratic function passes through the points (1, 0), (2, 5), and (3, 12). Sketch its graph. Then work with your team to develop an algebraic method to find the equation $y = ax^2 + bx + c$ of this specific quadratic function.

Discussion Points

What does the graph of any quadratic function look like?

What does it mean for the graph of $y = ax^2 + bx + c$ to pass through the point (3, 12)?

What solving method can we use to find a , b , and c ?

How can we check our equation?

Would this method allow us to find the equation of a quadratic using *any* three points?

Would this method work if we only had two points?

Further Guidance

6-62. How many points does it take to determine the equation of a linear function $y = ax + b$? Discuss this with your team and include at least one sketch to support your answer.

Now think about the graph of a quadratic function $y = ax^2 + bx + c$. How many points do you think it would take to determine this graph? Why? Does there need to be any restriction on the points you use? Discuss these questions with your team and **justify** your answers before moving on to part (a).

- a. Suppose you wanted the graph of a quadratic function $y = ax^2 + bx + c$ to pass through the points (1, 0), (2, 5), and (3, 12). How would these points be useful in finding the specific equation of this function? If your team has not already done so, include a sketch of the parabola going through these points to support

your answer.

- b. It is often useful to label points with the variable they represent. For instance, for the point (3, 12), which variable does the 3 represent? Which variable does the 12 represent?
- c. Using the general equation of a quadratic, $y = ax^2 + bx + c$, substitute the x - and y -values from your first point into the equation. Then do the same for the other two points to create three equations where the unknowns are a , b , and c .
- d. Now use the strategies you developed in Lesson 6.1.4 to solve the system of equations for a , b , and c .
- e. Use your results to write the equation of the quadratic function that passes through the points (1, 0), (2, 5), and (3, 12). How can you check your answer? That is, how can you make sure your equation would actually go through the three points? Using the method your team decides on, check your equation.

===== *Further Guidance* =====
section ends here.

6-63. LEARNING LOG

In your Learning Log, summarize the method you used in problem 6-61 (or problem 6-62) to find the equation $y = ax^2 + bx + c$ of the quadratic function whose graph passes through three given points. Title this entry “Finding the Equation of a Parabola Given Three Points” and label it with today's date.

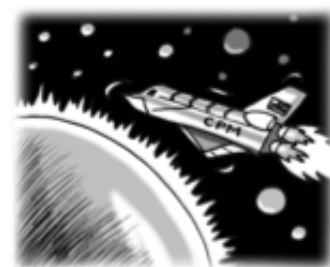


6-64. Find the equation $y = ax^2 + bx + c$ of the function that passes through the three points given in parts (a) and (b) below. Be sure to check your answers.

- a. (3, 10), (5, 36), and (-2, 15)
- b. (2, 2), (-4, 5), and (6, 0)

6-65. What happened in part (b) of problem 6-64? Why did this occur? (If you are not sure, plot the points on graph paper.)

6-66. CPM engineers are considering developing a private space rocket. In a computer simulation, the rocket is approaching a star and is caught in its gravitational pull. When the rocket's engines are fired, the rocket will slow down, stop momentarily, and then pick up speed and move away from the star, avoiding its gravitational field. CPM engaged the rocket engines when it was 750 thousand miles from the star. After one full minute, the rocket was 635 thousand miles from the star. After two minutes, the ship was 530 thousand miles from the star.



- a. Name the three points given in the information above if x = the time since the engines were engaged and y = the distance (in thousands of miles) from the star.
- b. Based on the points in part (a), make a rough sketch of a graph that shows the distance reaching a minimum and then increasing again, over time. What kind of function could follow this pattern?

- c. Find the equation of a graph that fits the three points you found in part (a).
- d. If the ship comes within 50 thousand miles of the star, the shields will fail and the ship will burn up. Use your equation to determine whether the space ship has failed to escape the gravity of the star.

6-67. Sickly Sid has contracted a serious infection and has gone to the doctor for help. The doctor takes a blood sample and finds 900 bacteria per cc (cubic centimeter) and gives Sid a shot of a strong antibiotic. The bacteria will continue to grow for a period of time, reach a peak, and then decrease as the medication succeeds in overcoming the infection. After ten days, the infection has grown to 1600 bacteria per cc. After 15 days it has grown to 1875.



- a. Name three data points given in the problem statement.
- b. Make a rough sketch that will show the number of bacteria per cc over time.
- c. Find the equation of the parabola that contains the three data points.
- d. Based on the equation, how long will it take until the bacteria are eliminated?
- e. Based on the equation, how long had Sid been infected before he went to the doctor?

6-68. THE COMMUTER

Sensible Sally has a job that is 35 miles from her home and needs to be at work by 8:15 a.m. She wants to get as much sleep as she can, leave as late as possible, and still get to work on time. Sally discovered that if she leaves at 7:10, it takes her 40 minutes to get to work. If she leaves at 7:30, it takes her 60 minutes to get to work. If she leaves at 7:40, it takes her 50 minutes to get to work. Since her commute time increases and then decreases, Sally decided to use a parabola to model her commute, assuming the time it takes her to get to work varies quadratically with the number of minutes after 7:00 that she leaves her house.



- a. If x = the number of minutes after 7:00 that Sally leaves, and y = the number of minutes it takes Sally to get to work, what three ordered pairs can you determine from the problem?
- b. Use the three points from part (a) to find the equation of a parabola in standard form that can be used to model Sally's commute.
- c. Will Sally make it to work on time if she leaves at 7:20?

6-69. PAIRS PARABOLA CHALLENGE

Your challenge will be to work with a partner to create a parabola puzzle for another pair of students to solve. Follow the directions below to create a puzzle that will make them think and allow them to show off their algebra skills. When you are ready, you will trade puzzles with another pair and attempt to solve theirs.

- a. With your partner, decide on an equation for a parabola and then identify three points that lie on its graph.

Keep track of how you came up with your equation and how you chose your points. Be ready to share strategies.

- b. Write the coordinates of the three points on an index card or small slip of paper to give to another pair of students. Be sure you keep a copy of your equation so you can check their work later.
- c. Trade points with another pair and work with your partner to solve their puzzle. When you are confident of your equation, check your work with the pair that wrote it.

6-70. Make a conjecture about how you would find the equation of a cubic function that passes through a given set of points when graphed, $y = ax^3 + bx^2 + cx + d$. How many points do you think you would need to be given to be able to determine a unique equation? How could you extend the method you developed for solving a quadratic to solving a cubic?



6-71. Solve the system of equations below and then check your solution in each equation. Be sure to keep your work well organized. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

$$x - 2y + 3z = 8$$

$$2x + y + z = 6$$

$$x + y + 2z = 12$$

6-72. Find the equation in $y = ax^2 + bx + c$ form of the parabola that passes through the points (1, 5), (3, 19), and (-2, 29). [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

6-73. This problem is a checkpoint for multiplying and dividing rational expressions. It will be referred to as Checkpoint 6A.



Multiply or divide each pair of rational expressions. Simplify the result.

a. $\frac{x^2-16}{(x-4)^2} \cdot \frac{x^2-3x-18}{x^2-2x-24}$

b. $\frac{x^2-1}{x^2-6x-7} \div \frac{x^3+x^2-2x}{x-7}$

Check your answers by referring to the [Checkpoint 6A materials](#).

If you needed help solving these problems correctly, then you need more practice. Review the [Checkpoint 6A materials](#) and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

6-74. Simplify each expression in parts (a) through (c) below. Then complete part (d). [Help \(Html5\)](#) \Leftrightarrow [Help](#)

(Java)

a. $xy(\frac{1}{x} + \frac{1}{2y})$

b. $ab(\frac{2}{a} + \frac{4a}{b})$

c. $2x(3 - \frac{1}{2x})$

d. What expression would go in the box to make the equation $\square(\frac{2}{x} + \frac{7}{y}) = 2y + 7x$ true?

6-75. Change each of the following equations from logarithmic form to exponential form, or vice versa. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

a. $y = \log_{12}x$

b. $x = \log_y 17$

c. $y = 1.75^{2x}$

d. $3y = x^7$

6-76. Solve $\sqrt{3x-6} + 6 = 12$ and check your solution. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

6-77. The half-life of an isotope is 1000 years. A 50-gram sample of the isotope is sealed in a box. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

- How much is left after 10,000 years?
- How long will it take to reduce to 1% of the original amount?
- How long will it take until all of the original sample of the isotope is gone? Support your answer.

6-78. Graph the following piece-wise defined function. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

$$x > 0; \quad y = |x| + 3$$

$$x \leq 0; \quad y = x^3 + 3$$

- Now shift the function down 3 and to the left 2 and draw the new graph on the same set of axes.
- Write the new equations for the shifted function.

6-79. Rewrite each expression below as an exponential expression with a base of 2. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

a. 16

- b. $\frac{1}{8}$
- c. $\sqrt{2}$
- d. $\sqrt[3]{4}$

6-80. Solve the system of equations below and then check your solution in each equation. Be sure to keep your work well organized. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

$$\begin{aligned}x + 2y - z &= -1 \\2x - y + 3z &= 13 \\x + y + 2z &= 14\end{aligned}$$

6-81. Find an equation for the parabola that passes through the points $(-1, 10)$, $(0, 5)$, and $(2, 7)$. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

6-82. Change each of the following equations from logarithmic form to exponential form, or vice versa. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

- a. $a = \log_b 24$
- b. $3x = \log_{2y} 7$
- c. $3y = 2^{5x}$
- d. $4p = (2q)^6$

6-83. Add or subtract and simplify each of the following expressions. Justify that each step of your process makes sense. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

- a. $\frac{3x}{x^2+2x+1} + \frac{3}{x^2+2x+1}$
- b. $\frac{3}{x-1} - \frac{2}{x-2}$

6-84. On their Team Test, Raymond, Sarah, Hannah, and Aidan were given $y = 4x^2 - 24x + 7$ to change into graphing form. Raymond noticed that the leading coefficient was a 4 and not a 1. His team agreed on a way to start rewriting, but then they worked in pairs and got two different solutions, shown below. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)



Raymond and Hannah

- (1) $y = 4x^2 - 24x + 7$
- (2) $y = 4(x^2 - 6x) + 7$
- (3) $y = 4(x^2 - 6x + 3^2) + 7 - 36$
- (4) $y = 4(x - 3)^2 - 29$

Aidan and Sarah

- (1) $y = 4x^2 - 24x + 7$
- (2) $y = 4(x^2 - 6x + 9) + 7 - 9$
- (3) $y = 4(x - 3)^2 + 7 - 3^2$
- (4) $y = 4(x - 3)^2 - 2$

Hannah says, “*Aidan and Sarah made a mistake in Step 3. Because of the factored 4 they really added $4(9)$ to complete the square, so they should subtract 36, not just 9.*” Is Hannah correct? **Justify** your answer by showing whether the results are equivalent to the original equation.

6-85. Use the correct method from problem 6-84 to change each of the following equations to graphing form. Then, without graphing, find the vertex and equation of the line of symmetry for each. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

a. $y = 2x^2 - 8x + 7$

b. $y = 5x^2 - 10x - 7$

6-86. Shift the graph of $\log x$ up 3 units and to the right 6. Graph both the original and the transformed graph on the same set of axes and write the equation for the transformed graph. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

6-87. Given $f(x) = 2x^2 - 4$ and $g(x) = 5x + 3$, find the value of each expression below. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

a. $f(a)$

b. $f(3a)$

c. $f(a + b)$

d. $f(x + 7)$

e. $f(5x + 3)$

f. $g(f(x))$