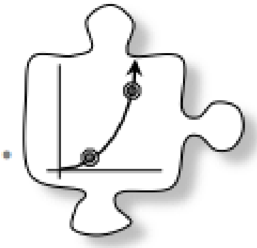
6.2.1 How can I solve exponential equations?



Using Logarithms to Solve Exponential Equations

In Chapter 5, you learned what a logarithm was and several important facts about logs. In this lesson, you will learn about a property of logarithms that will be very useful for solving problems that involve exponents.

6-88. LOGARITHMS SO FAR

There are three important log facts you have learned so far. Discuss these questions with your team to ensure everyone remembers these ideas. For each problem, make up an example to illustrate your ideas.

a. What is a logarithm? How can log equations be converted into another form?

b. What do you know about the logarithm key on your calculator?

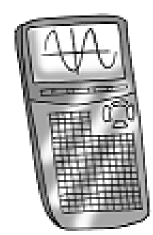
c. What does the graph of $y = \log(x)$ look like? Write a general equation for $y = \log(x)$.

6-89. Marta was convinced that there had to be some way to graph $y = \log_2 x$ on her graphing calculator. She typed in $y = \log(2^x)$ and pressed GRAPH.

"*It WORKED!*" Marta yelled in triumph.

"*Whaaaat?*" said Celeste. "*I think $y = \log_2 x$ and $y = \log(2^x)$ are totally different, and I bet we can prove it by converting both of them to exponential form.*"

"*Yeah, I think you're wrong, Marta,*" said Sophia. "*I think we can show that $y = \log_2 x$ and $y = \log(2^x)$ are totally different by looking at the graphs.*"



a. Show that the two equations are different by sketching the graph of $y = \log_2 x$. Then sketch what your graphing calculator shows to be the graph of $y = \log(2^x)$.

b. Now show that $y = \log_2 x$ and $y = \log(2^x)$ are different by converting both of them to exponential form.

6-90. The work you did in problem 6-89 is a **counterexample**, which shows that in general, the statement $\log_b x = \log(b^x)$ is *false*. For each of the following log statements, use the strategies from problem 6-89 to determine whether they are true or false, and justify your answer. Be ready to present your conclusions and justifications.

a. $\log_5(25) \stackrel{?}{=} \log_{25}(5)$

b. $\log(x^2) \stackrel{?}{=} (\log x)^2$

c. $\log(7^x) \stackrel{?}{=} x\log(7)$

d. $\log(2x) \stackrel{?}{=} \log_2 x$

6-91. In the previous problem only *one* of the statements was true.

a. Use different numbers to make up four more statements that follow the same pattern as the one true statement, and test each one to see whether it appears to be true.

b. Use your results to complete the following statement, which is known as the **Power Property of Logs**: $\log(b^x) =$ ______.

6-92. Do you remember solving problems like $1.04^x = 2$ in your homework? What method(s) did you and your teammates use to find x? In tonight's homework there are several more of these problems. (You probably wish there were a more efficient way!)

6-93. THERE MUST BE AN EASIER WAY

It would certainly be helpful to have an easier method than Guess and Check to solve equations like $1.04^x = 2$. Complete parts (a) through (c) below to discover an easier way.

a. What makes the equation $1.04^x = 2$ so hard to solve?

b. Surprise! In the first part of this lesson, you already found a method for getting rid of inconvenient exponents! Talk with your team about how your results from problems 6-90 and 6-91 can help you rewrite the equation $1.04^x = 2$. Be prepared to share your ideas with the class.

c. Solve $1.04^x = 2$ using this new method. Be sure to check your answer.

6-94. Solve the following equations. After checking your answer, round them to three decimal places.

a. $5 = 2.25^x$

b. $3.5^x = 10$

c. $2(8^x) = 128$

d. $2x^8 = 128$

Review & Preview

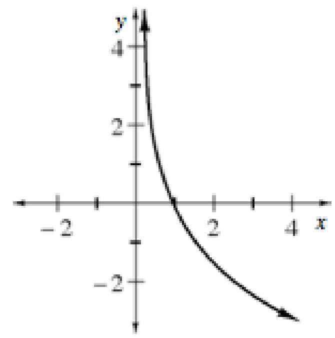
6-95. Complete the table at right and find its equation. Help (Html5)⇔Help (Java)

x	y
	1
	3
2	9
	27
4	
	243
6	
7	
8	

6-96. Margee thinks she can use logs to solve $56 = x^8$, since logs seem to make exponents disappear. Unfortunately, Margee is wrong. Explain the difference between equations like $2 = 1.04^x$, in which you can use logs, and $56 = x^8$, in which it does not make sense to use logs. Help (Html5)⇔Help (Java)

6-97. What values must x have so that $\log(x)$ is greater than 2? Justify your answer. Help (Html5)⇔Help (Java)

6-98. At right is a graph of $y = \log_b x$. Describe the possible values for b. Help (Html5)⇔Help (Java)

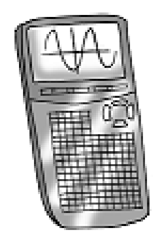


6-99. Consider the questions below. Help (Html5)⇔Help (Java)

a. What can you multiply 8 by to get 1?

b. What can you multiply x by to get 1?

c. Using the rules of exponents, find a way to solve $m^8 = 40$. Remember that logarithms will not be useful here, but the exponent key on your calculator *will* be. (Obtain the answer as a decimal approximation using your calculator. Check your result by raising it to the 8^{th} power.)



d. Now solve $n^6 = 300$.

e. Describe a method for solving $x^a = b$ for x with a calculator.

6-100. Adam keeps getting negative exponents and fractional exponents confused. Help him by explaining the difference between $2^{1/2}$ and 2^{-1}. Help (Html5)⇔Help (Java)

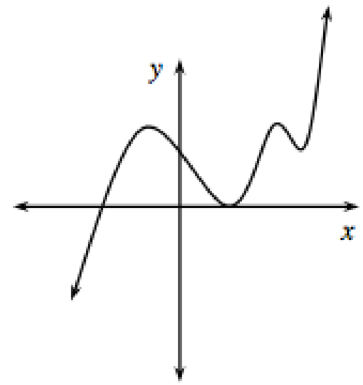
6-101. Solve each inequality and graph its solution on a number line. Help (Html5)⇔Help (Java)

a. $|x| < 3$

b. $|2x+1| < 3$

c. $|2x+1| \geq 3$

6-102. Consider the graph at right. Help (Html5)⇔Help (Java)



a. Is the graph a function? Explain.

b. Make a sketch of the inverse of this graph. Is the inverse a function? Justify your answer.

c. Must the inverse of a function be a function? Explain.

d. Describe what is characteristic about functions that do have inverse functions.

e. Could the inverse of a non-function be a function? Explain or give an example.

6-103. Solve each system of equations below. Help (Html5)⇔Help (Java)

a. $-4x = z - 2y + 12$
$y + z = 12 - x$
$8x - 3y + 4z = 1$

b. $3x + y - 2z = 6$
$x + 2y + z = 7$
$6x + 2y - 4z = 12$

c. What does the solution in part (b) tell you about the graphs?