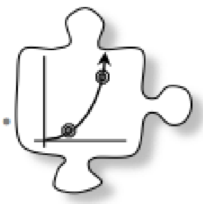
6.2.2 How can I rewrite it?



Investigating the Properties of Logarithms

You already know the basic rules for working with exponents. Since logs are the inverses of exponential functions, they also have properties that are similar to the ones you already know. In this lesson, you will explore these properties.

6-104. Marta now knows that if she wants to find $\log_2(30)$, she cannot just type $\log(2^{30})$ into her calculator, since her calculator's log key cannot directly calculate logs with base 2. But she still wants to be able to find what $\log_2(30)$ equals.

a. First, use your knowledge of logs to estimate $\log_2(30)$.

b. Now use what you learned in lesson 6.2.1 to get a better estimate. Since you want to determine what $\log_2(30)$ equals, you can write $\log_2(30) = x$. When working with a log equation, it is often easier to first convert it to exponential form. Rewrite this equation in exponential form.

c. Use the methods you developed in class to solve this equation. Refer back to your work on problem 6-93 if you need help.

6-105. Congratulations! You are smarter than your calculator. You have just evaluated a log with base 2, even though your calculator does not do that. Now you will practice some more.

a. First estimate an answer, then apply the method you have just developed to evaluate $\log_5(200)$.

b. Apply the process you used in part (a) to evaluate the expression $\log_a b = x$.

6-106. Since logs and exponentials are inverses, the properties of exponents (which you already know) also apply to logs. The problems below will help you discover these new log properties.

a. Complete the two exponent rules below. In part (b), you will find the equivalent properties of logs.

$x^a x^b =$ ____ and $\frac{x^b}{x^a} =$ ____

b. To help you find the equivalent log properties, use your calculator to solve for x in each problem below. Note that x is a whole number in parts (i) through (v). Look for patterns that would make your job easier and allow you to generalize in part (vi).

i. $\log(5) + \log(6) = \log(x)$

ii. $\log(5) + \log(2) = \log(x)$

iii. $\log(5) + \log(5) = \log(x)$

iv. $\log(10) + \log(100) = \log(x)$

v. $\log(9) + \log(11) = \log(x)$

vi. $\log(a) + \log(b) = \log($____$)$

c. What if the log expressions are being subtracted instead of added? Solve for x in each problem below. Note that x will not always be a whole number. Again, look for patterns that will allow you to generalize in part (vi).

i. $\log(20) - \log(5) = \log(x)$

ii. $\log(30) - \log(3) = \log(x)$

iii. $\log(5) - \log(2) = \log(x)$

iv. $\log(17) - \log(9) = \log(x)$

v. $\log(375) - \log(17) = \log(x)$

vi. $\log(b) - \log(a) = \log(\_\_\_\_)$

6-107. LEARNING LOG

The two properties you found in problem 6-106 work for logs in any base, not just base 10. (You will officially prove this later.) You now know three different log properties and you have developed a process for solving log problems that are not in base 10. Write and explain each of these log properties in your Learning Log. Be sure to include examples, with at least one problem where you need to change to base 10. Title this entry "Logarithm Properties" and label it with today's date.

6-108. LOG PROPERTY PUZZLES
Obtain the Lesson 6.2.2A Resource Page from your teacher or copy the table below. Use the log properties to fill in the missing parts. Be sure to remember that in every row, each expression is equivalent to every other expression.

		Product Property				Quotient Property		
$\log_3 60$	=	$\log_3 6 + \_\_\_$	=	$\log_3 3 + \_\_\_$	=	$\log_3 120 - \_\_\_$	=	$\log_3 240 - \_\_\_$
$\log_7 36$	=		=		=		=	
	=	$\log_6 9 + \log_6 2$	=		=		=	
	=		=		=	$\log_{25} 75 - \log_{25} 1.5$	=	
	=		=		=	$\log 160 - \log 4$	=	

6-109. Use the properties of logs to write each of the following expressions as a single logarithm, if possible.

a. $\log_{1/2}(4) + \log_{1/2}(2) - \log_{1/2}(5)$

b. $\log_2(M) + \log_3(N)$

c. $\log(k) + x\log(m)$

d. $\frac{1}{2}\log_5 x + 2\log_5(x+1)$

e. $\log(4) - \log(3) + \log(\pi) + 3\log(r)$

f. $\log(6) + 23$

6-110. What values must x have so that $\log(x)$ has a negative value? Justify your answer.

6-111. The fact that for any base m (when $m > 0$), $\log_m a + \log_m b = \log_m ab$ is called the **Product Property of Logarithms**. To prove that this property is true, follow the directions below.

a. Since logarithms are the inverses of exponential functions, each of their properties can be derived from a similar property of exponents. Here, you are trying to prove that "logs turn products into sums." First, recall similar properties of exponents. If $a = m^x$ and $b = m^y$, write $a \cdot b$ as a power of m.

b. Rewrite $a = m^x$, $b = m^y$, and your answer to part (a) in logarithmic form.

c. In the third equation you wrote for part (b), substitute for x and y to obtain a log equation of base m that involves only the variables a and b.

d. The property $\log_m a - \log_m b = \log_m \frac{a}{b}$ is called the **Quotient Property of Logarithms**. Use $a = m^x$ and $b = m^y$ to express $\frac{a}{b}$ as a power of m. Then use a similar process to rewrite each into log form and prove the Quotient Property of Logs.

6-112. The **Power Property of Logarithms**, which you learned in Lesson 6.2.1, is a little trickier to prove. A proof is given below. As you copy each step onto your paper, work with your team to make sense of what was done. Give a reason for each step.

To prove that $\log_m a^n = n \log_m a$,

Let $\log_m a^n = p$ and $n \log_m a = q$

Convert to $m^p = a^n$

First rewrite as $\log_m a = \frac{q}{n}$ and then convert to $m^{q/n} = a$.

Using the two resulting equations, substitute for a and then simplify:

$$m^p = (m^{q/n})n$$
$$m^p = m^q$$

Therefore, $p = q$.

Remember that $p = \log_m a^n$ and $q = n \log_m a$, so $\log_m a^n = n \log_m a$, which was the goal of the proof.

METHODS AND MEANINGS

MATH NOTES

Logarithm Properties

The following definitions and properties hold true for all positive $m \neq 1$.

Definition of logs:	$\log_m (a) = n$ means $m^n = a$
Product Property:	$\log_m (a \cdot b) = \log_m (a) + \log_m (b)$
Quotient Property:	$\log_m (\frac{a}{b}) = \log_m (a) - \log_m (b)$
Power Property:	$\log_m (a^n) = n \cdot \log_m (a)$
Inverse relationship:	$\log_m (m)^n = n$ and $m^{\log_m (n)} = n$

Review & Preview

6-113. Solve each of the following equations to the nearest 0.001. Help (Html5)⇔Help (Java)

a. $(5.825)^{(x-3)} = 120$

b. $18(1.2)^{(2x-1)} = 900$

6-114. Simplify each expression below. If you are stuck, the ideas in problem 6-74 should be helpful. Help (Html5)⇔Help (Java)

a. $\dfrac{x}{1-\frac{1}{x}}$

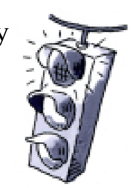
b. $\dfrac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{b}-a}$

6-115. Use the definition of a logarithm to change $\log_2 7$ into a logarithmic expression of base 5. Help (Html5)⇔Help (Java)

6-116. Sketch the graph of $y = \log_3(x + 4)$ and describe the transformation from its parent graph. Help (Html5)⇔Help (Java)

6-117. Due to the worsened economy, merchants in downtown Hollywood cannot afford to replace their outdoor light bulbs when the bulbs burn out. On average, about thirteen percent of the light bulbs burn out every month. Assuming there are now about one million outside store lights in Hollywood, how long will it take until there are only 100,000 bulbs lit? Until there is only one bulb lit? Help (Html5)⇔Help (Java)

6-118. Raymond, Hannah, Aidan, and Sarah were working together to change $y = 3x^2 - 15x - 5$ into graphing form. They started by rewriting it as $y = 3(x^2 - 5x) - 5$, when Raymond said, "*Will this one work? Look, the perfect square would have to be* $(x - 2.5)^2$.



After thinking about it for a while, Sarah said, "*That's OK. Negative 2.5 squared is 6.25, but because of the 3 we factored out, we are really adding* 3(6.25)."

"*Yes,*" Aidan added, "*So we have to subtract 18.75 to get an equivalent equation.*"

Hannah summarized with the work shown at right.

$$y = 3x^2 - 15x - 5$$
$$y = 3(x^2 - 5x) - 5$$
$$y = 3(x - 2.5)^2 - 5 - 18.75$$
$$y = 3(x - 2.5)^2 - 23.75$$

What do you think? Did they rewrite the equation correctly? If so, find the vertex and the line of symmetry of the parabola. If not, explain their mistakes and show them how to do it correctly. Help (Html5)⇔Help (Java)

6-119. Use the ideas developed in problem 6-118 to change each of the following quadratic equations into graphing form. Identify the vertex and the line of symmetry for each one. Help (Html5)⇔Help (Java)

a. $f(x) = 4x^2 - 12x + 6$

b. $g(x) = 2x^2 + 14x + 4$

6-120. Consider the function $y = 3(x + 2)^2 - 7$ as you complete parts (a) through (c) below. Help (Html5)⇔Help (Java)

a. How could you restrict the domain to show 'half' of the graph?

b. Find the equation for the inverse function for your “half” graph.

c. What are the domain and range for the inverse function?

6-121. Add or subtract and simplify each of the following expressions. Justify that each step of your process makes sense. Help (Html5)⇔Help (Java)

a. $\frac{3}{(x-4)(x+1)}+\frac{6}{x+1}$

b. $\frac{x+2}{x^2-9}-\frac{1}{x+3}$

6-122. Eniki has a sequence of numbers given by the formula $t(n) = 4(5^n)$. Help (Html5)⇔Help (Java)

a. What are the first three terms of Eniki's sequence?

b. Chelita thinks the number 312,500 is a term in Eniki's sequence. Is she correct?**Justify** your answer by either giving the term number or explaining why it is not in the sequence.

c. Elisa thinks the number 94,500 is a term in Eniki's sequence. Is she correct? Explain.