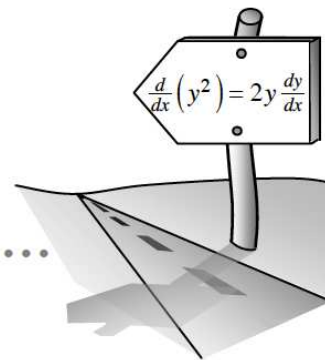# 6.2.2 How do I use the Chain Rule to implicitly differentiate?

Implicit Differentiation Practice

6-67. Implicit differentiation enables us to find $\frac{dy}{dx}$ for *any* relation involving x and y, even if y cannot be solved explicitly! For example, consider the relation: $y^3 + y - 2x = 5$.

   a. If possible, solve this equation for y in terms of x. If not possible, explain why not.

   b. Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

   c. If possible, find any values of x for which this relation is non-differentiable.

   d. Does this relation have any horizontal tangents? Use $\frac{dy}{dx}$ to support your claim.

   e. Find the slopes of the tangent at the points $(2.5, 2)$ and $(-1.5, 1)$.

   f. Find $\frac{d^2y}{dx^2}$. Simplify your answer so that only y and x are present.

6-68. RADICAL RELATIONS, Part One

The equation $x^3 + y^3 = 9xy$ is called a "Folium of Descartes." Its graph is shown below.
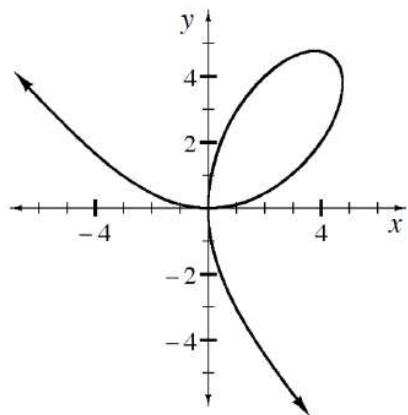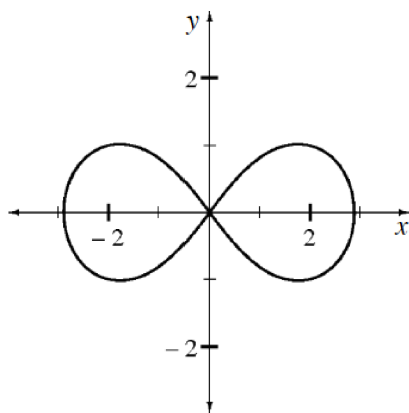

   a. Estimate its slope at $(4, 2)$. Then find the exact slope at $(4, 2)$ algebraically.

   b. Is the derivative of this Folium a function? Explain.

6-69. RADICAL RELATIONS, Part Two

A "lemniscate" looks like an infinity symbol. The lemniscate graphed below has the equation:
$3(x^2 + y^2)^2 = 25(x^2 - y^2)$

   a. Algebraically find the slope(s) of the tangent(s) to the lemniscate at $x = 2$.

   b. Not only is the derivative of this lemniscate *not* a function, it does not exist at two places. Explain.

6-70. THE SECOND DERIVATIVE OF A CIRCLE

Consider the circle $x^2 + y^2 = 16$.

   a. Find and simplify $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Determine the best strategy to do this.

   b. Compare the second derivative to the graph of $x^2 + y^2 = 16$. In which quadrants is the graph of the circle concave up? Concave down? Where is the slope changing least rapidly?

Review & Preview

6-71. Given the equation $x^2 - 2xy + \tan y = 4$, find an expression for $\frac{dy}{dx}$.   Homework Help ✎

6-72. ASTROID

The graph of $x^{2/3} + y^{2/3} = 4$ is called an "astroid."   Homework Help ✎
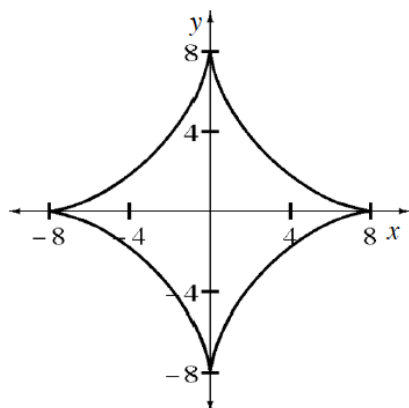
a. Use implicit differentiation to find $\frac{dy}{dx}$.

b. Where does the astroid have a horizontal tangent? Explain.

c. Where is the derivative undefined? What happens at those points?

6-73. Below is the number of gold medals the U.S. has received in the last ten Summer Olympic Games. Find the mean number (average) of gold medals the U.S. wins. Write a brief statement describing how you found the mean.   Homework Help ✎
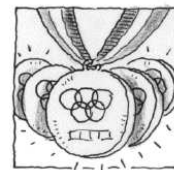
Year	# of Gold Medals	Year	# of Gold Medals
1954	21	1976	31
1960	17	1980	55
1964	35	1984	37
1968	42	1992	41
1972	39	1996	48

6-74. From her second floor window 6 meters above the ground, Xiomara throws a tennis ball at a rate of 11 meters per second up towards her friend Itzagueri who is 12 meters above the ground.
Assume $a(t) = -9.8$ meters per sec^2 for the acceleration due to gravity.   Homework Help ✎
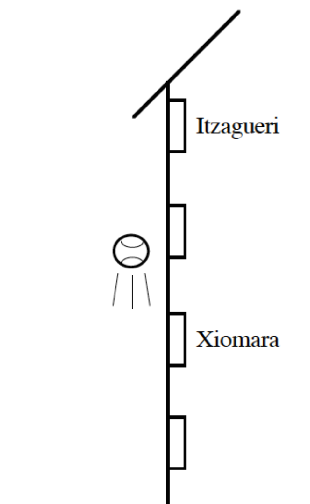
a. Assuming Itzagueri does not catch the ball, describe the motion of the ball 2 seconds after it was thrown.

b. When does the ball reach its highest point?

c. Will the ball reach Itzagueri? Support your answer with calculations.

d. Determine how high the ball is when it is falling at the rate of 5 meters per second.

e. Explain what $\int_0^2 a(t)\,dt$ represents physically.

6-75. While riding his bike to a pond, Steven's distance in miles was modeled by $s(t)$ below. If the lake was 9 miles away and if t is measured in hours: Homework Help ✎

$$s(t) = t^3 - 3t^2 + 3t$$

a. What was Steven's maximum velocity during the trip? When did it occur?

b. Did Steven ever stop during the trip? Justify your conclusion analytically.

c. What was Steven's average velocity?

6-76. The good news for the small prairie town of Woisme is that a new fiber optics plant is about to open. The bad news is that the old quarry is about to close, and many residents will lose their jobs as a result. The population is predicted to follow this model over the next few years: Homework Help ✎

$$p(t) = 100(4 \cdot 2^t - 3^t) = 400 \cdot 2^t - 100 \cdot 3^t.$$

a. Using complete sentences, relate the equation for $p(t)$ to the changes of industry.

b. Algebraically determine when Woisme will have the most people. Use your calculator only for numerical calculations.
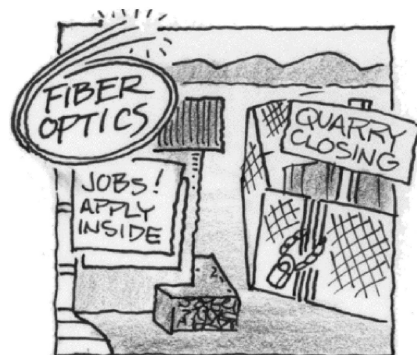
6-77. The graph of $y = \frac{1}{x}$ is shown below. The tangent line at $(2, 0.5)$, along with the coordinate axes, form a shaded triangle.   Homework Help ✎
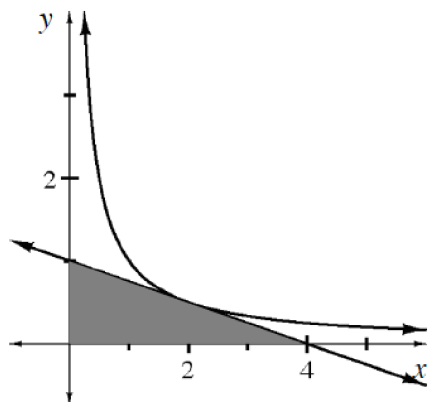
a. Find the equation of the tangent line.

b. Explain why the tangent line will always give an under approximation of the curve using the second derivative.

c. Find the area of the shaded triangle.

d. Find the area of the triangle formed when the tangent is instead placed at $(1, 1)$.

e. Prove that the area of the shaded triangle formed by a tangent to $y = \frac{1}{x}$ is always the same, regardless of the point of tangency.

6-78. For each of the following functions, find $\frac{dy}{dx}$.   Homework Help ✎

a. $y = 3e^{3x}$

b. $x^2 \frac{dy}{dx} - 3y = 2 \frac{dy}{dx}$

c. $y = \int_2^x (\ln t^2 - 4)\, dt$

d. $y \cos x - 4x = 8$