

6.2.3 How can I find an exponential function?

Writing Equations of Exponential Functions



You have worked with exponential equations throughout this chapter. Today you will look at how you can find the equation for an exponential function using data.

6-123. DUE DATE

Brad's mother has just learned that she is pregnant! Brad is very excited that he will soon become a big brother. However, he wants to know when his new sibling will arrive and decides to do some research. On the Internet, he finds the following article:

Hormone Levels for Pregnant Women

When a woman becomes pregnant, the hormone HCG (human chorionic gonadotropin) is produced to enable the baby to develop.

During the first few weeks of pregnancy, the level of HCG hormone grows exponentially, starting with the day the embryo is implanted in the womb. However, the rate of growth varies with each pregnancy. Therefore, doctors cannot use just a single test to determine how long a woman has been pregnant. They must test the levels over time. Commonly, the HCG levels are measured two days apart to look for this rate of growth.

Brad's mother says she was tested for HCG during her last two doctor visits. On March 21, her HCG level was 200 mIU/ml (milli-international units per milliliter). Two days later, her HCG level was 392 mIU/ml.

- Assuming that the model for HCG levels is of the form $y = ab^x$, find an equation that models the growth of HCG for Brad's mother's pregnancy.
- Assuming that Brad's mother's level of HCG on the day of implantation was 5 mIU/ml, on what day did the baby most likely become implanted? How many days after implantation was his mother's first doctor visit?
- Brad learned that a baby is born approximately 37 weeks after implantation. When can Brad expect his new sibling to be born?

6-124. SOLVING STRATEGIES

In problem 6-123, you and your team developed a strategy to find the equation of an exponential equation of the form $y = ab^x$ when given two points on the curve.

- a. What different strategies were generated by the other teams in your class? If no one shares your solving method with the class, be sure to share yours. Take notes on the different strategies that are presented.
- b. Did any team use a system of exponential equations to solve for a and b ? If not, examine this strategy as you answer the questions below.
 - i. The doctor visits provide two data points that can help you find an exponential model: (21, 200) and (23, 392). Use each of these points to substitute for x and y into $y = ab^x$. You should end up with two equations in terms of a and b .
 - ii. Consider the strategies you already have for solving systems of equations. Are any of those strategies useful for this problem? Discuss a way to solve your system from part (i) for a and b with your team. Be ready to share your method with the class.

6-125. The context in problem 6-123 required you to assume that the exponential model had an asymptote at $y = 0$ to find the equation of the model. But what if the asymptote is not at the x -axis? Consider this situation below.

- a. Assume the graph of an exponential function passes through the points (3, 12.5) and (4, 11.25). Is the exponential function increasing or decreasing? Justify your answer.
- b. If the horizontal asymptote for this function is the line $y = 10$, make a sketch of its graph showing the horizontal asymptote.
- c. If this function has the equation $y = ab^x + c$, what would be the value of c ? Use what you know about this function to find its equation. Verify that as x increases, the values of y get closer to $y = 10$.
- d. Find the y -intercept of the function. What is the connection between the y -intercept and the asymptote?

6-126. Janice would like to have \$40,000 to help pay for college in 8 years. Currently, she has \$1000. What interest rate, when compounded yearly, would help her reach her goal?

- a. What type of function would best model this situation? Explain how you know and write the general form of this function.
- b. If y represents the amount of money and x represents the number of years after today, find an equation that models Janice's financial situation. What interest rate does she need to earn?
- c. Janice's friend Sarah starts with \$7800 and wants to have \$18,400 twenty years from now. What interest rate does she need (compounded yearly)?
- d. Is Janice's goal or Sarah's goal more realistic? Justify your response.



6-127. Ryan has the chickenpox! He was told that the number of pockmarks on his body would grow exponentially until his body overcomes the illness. He found that he had 60 pockmarks on November 1, and by November 3 the number had grown to 135. To find out when the first pockmark appeared, he will need to find the exponential function that will model the number of pockmarks based on the day. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

- Ryan decides to find the exponential function that passes through the points (3, 135) and (1, 60). Use these points to write the equation of his function of the form $f(x) = ab^x$.
- According to your model, what day did Ryan get his first chickenpox pockmark?

6-128. Give an example of an equation that requires the use of logarithms to solve it. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

6-129. Write three different, but equivalent, expressions for each of the following logs. For example: $\log(7^{3/2})$ can be written as $\frac{3}{2}\log(7)$, $\frac{1}{2}\log(7^3)$, $3\log(\sqrt{7})$, etc. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

- $\log(8^{2/3})$
- $-2\log(5)$
- $\log(na)^{bo}$

6-130. Kendra just made a cup of hot chocolate that was too hot for her to drink. She set it aside so it could cool off. While she was waiting, her friend Lara called and Kendra forgot about her hot chocolate. Sketch a graph that shows the temperature of the hot chocolate since Kendra first set it aside. How cold will the hot chocolate get? [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

6-131. Find the equation of the parabola that passes through the points (-2, 24), (3, -1), and (-1, 15). [6-131 HW eTool](#) (Desmos). [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

6-132. Use $f(x) = 3 + \sqrt{2x-1}$ to complete parts (a) through (e) below. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

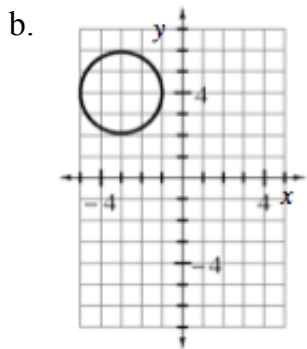
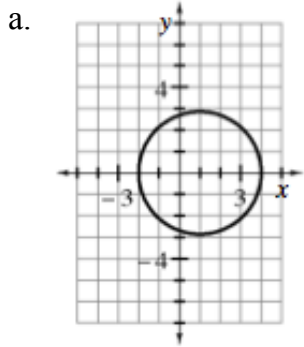
- What are the domain and range of $f(x)$?
- What is the inverse of $f(x)$? Call it $g(x)$.
- What are the domain and range of $g(x)$?
- Find an expression for $f(g(x))$.
- Find an expression for $g(f(x))$. What do you notice? Why does this happen?

6-133. Solve each of the following equations for x . [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

a. $x^3 = 243$

b. $3^x = 243$

6-134. Write the equation of each circle graphed below. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)



6-135. Add or subtract each expression below. Be sure to simplify. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

a. $\frac{x^2}{x-5} - \frac{25}{x-5}$

b. $\frac{a^2}{a+5} + \frac{10a+25}{a+5}$

c. $\frac{x^2}{x-y} - \frac{2xy-y^2}{x-y}$

d. $\frac{x}{x+1} + \frac{1}{x-1}$

6-136. Find the inverse of each of the functions below. Write your answers in function notation. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

a. $p(x) = 3(x^3 + 6)$

b. $k(x) = 3x^3 + 6$

c. $h(x) = \frac{x+1}{x-1}$

d. $j(x) = \frac{2}{3-x}$