

$$(2, 3) \quad (1, ?) \quad (0, ?) \rightarrow ax^2 + bx + c$$

6.1.

Cornell Notes:	Name:
	Assignment: 6.2.3 Writing Equations of Exponential Functions
	Class: Algebra II
Essential Question: How can I find an exponential function?	$(y = a \cdot b^x)$
	Notes

You have worked with exponential equations throughout this chapter. Today you will look at how you can find the equation for an exponential function using data.

6-123 DUE  
DATE

Brad's mother has just learned that she is pregnant! Brad is very excited that he will soon become a big brother. However, he wants to know when his new sibling will arrive and decides to do some research. On the Internet, he finds the following article:

### Hormone Levels for Pregnant Women

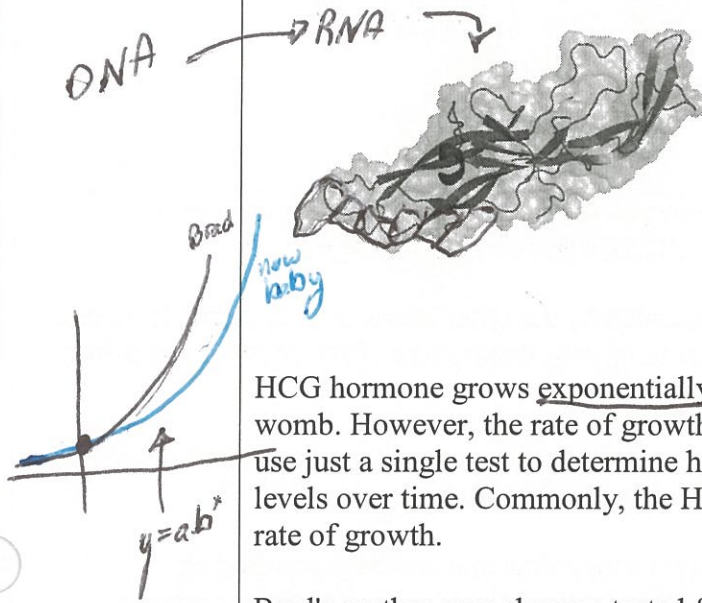
When a woman becomes pregnant, the hormone HCG (human chorionic gonadotropin) is produced to enable the baby to develop.

During the first few weeks of pregnancy, the level of

HCG hormone grows exponentially, starting with the day the embryo is implanted in the womb. However, the rate of growth varies with each pregnancy. Therefore, doctors cannot use just a single test to determine how long a woman has been pregnant. They must test the levels over time. Commonly, the HCG levels are measured two days apart to look for this rate of growth.

Brad's mother says she was tested for HCG during her last two doctor visits. On March 21, her HCG level was 200 mIU/ml (milli-international units per milliliter). Two days later, her HCG level was 392 mIU/ml.

a. Assuming that the model for HCG levels is of the form  $y = ab^x$ , find an equation that models the growth of HCG for Brad's mother's pregnancy.



$(0, 200)$   $(2, 392)$

time 0  
March 21

$x \rightarrow$  time  
 $y \rightarrow$  HCG levels

$$y = a \cdot b^x$$

$$200 = a \cdot b^0$$

$$200 = a$$



$$392 = a \cdot b^2$$

$$392 = 200 \cdot b^2$$

$$\frac{392}{200} = b^2$$

$$\sqrt{\frac{392}{200}} = b$$

$$1.4 = b$$

"plug in x and y"

"plug in  $a = 200$ "

"divide both sides by 200"

" $\sqrt{\quad}$  both sides"

$$y = 200 \cdot 1.4^x$$

HCG levels as a function  
of time relative March 21

"output"  $y$ , HCG

- b. Assuming that Brad's mother's level of HCG on the day of implantation was 5 mIU/ml, on what day did the baby most likely become implanted? How many days after implantation was his mother's first doctor visit?

$$y = 200 \cdot 1.4^x$$

$$5 = 200 \cdot 1.4^x$$

$$\left(\frac{5}{200}\right) = 1.4^x$$

$$\log\left(\frac{5}{200}\right) = \log(1.4^x)$$

$$\log\left(\frac{5}{200}\right) = x \cdot \log(1.4)$$

$$\frac{\log\left(\frac{5}{200}\right)}{\log(1.4)} = x$$

$$\frac{\log(0.025)}{\log(1.4)} = x$$

$$\frac{-1.602}{0.146} = x$$

$$-10.9 = x$$

HCG was 5  
10.9 days  
before  
March 21  
March 10

- c. Brad learned that a baby is born approximately 37 weeks after implantation. When can Brad expect his new sibling to be born?

Nov. 10

6-124

In problem 6-123, you and your team developed a strategy to find the equation of an exponential equation of the form  $y = ab^x$  when given two points on the curve.

- a. What different strategies were generated by the other teams in your class? If no one shares your solving method with the class, be sure to share yours. Take notes on the different strategies that are presented.

- b. Did any team use a system of exponential equations to solve for  $a$  and  $b$ ? If not, examine this strategy as you answer the questions below.

- i. The doctor visits provide two data points that can help you find an exponential model: (21, 200) and (23, 392). Use each of these points to substitute for  $x$  and  $y$  into  $y = ab^x$ . You should end up with two equations in terms of  $a$  and  $b$ .

$$y = a \cdot b^x$$

$$200 = a \cdot b^{21}$$

$$\frac{200}{b^{21}} = a$$

$$\frac{200}{(1.4)^{21}} = a$$

$$0.171 = a$$

$$392 = a \cdot b^{23}$$

$$392 = \frac{200}{b^{21}} \cdot b^{23}$$

$$392 = 200 \cdot b^2$$

$$\frac{392}{200} = b^2$$

$$1.4 = b$$

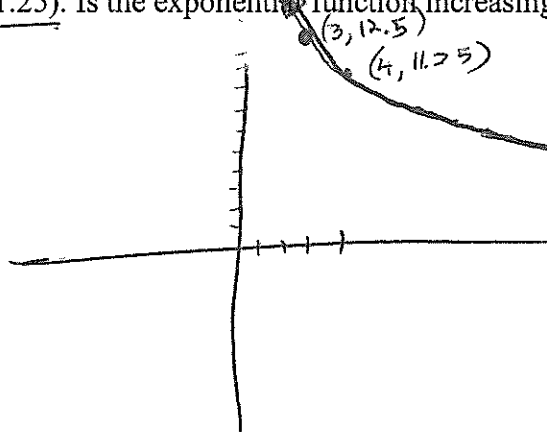
$$y = 0.171 \cdot 1.4^x$$

HCG levels relative to March 1<sup>st</sup>

what would  $x=0$   
be if march 21  
is  $x=21$

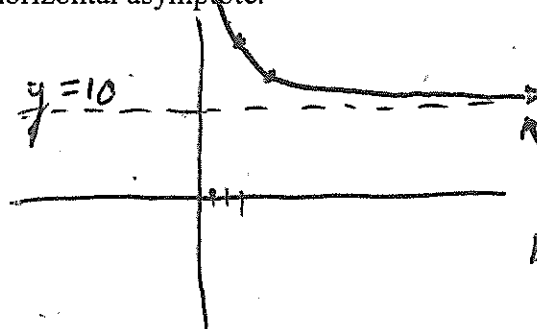
The context in problem 6-123 required you to assume that the exponential model had an asymptote at  $y = 0$  to find the equation of the model. But what if the asymptote is not at the  $x$ -axis? Consider this situation below.

- a. Assume the graph of an exponential function passes through the points  $(3, 12.5)$  and  $(4, 11.25)$ . Is the exponential function increasing or decreasing? Justify your answer.



Decreasing. As we look left to right the graph goes down.

- b. If the horizontal asymptote for this function is the line  $y = 10$ , make a sketch of its graph showing the horizontal asymptote.



get close to horizontal asymptote but do not touch

- c. If this function has the equation  $y = ab^x + c$ , what would be the value of  $c$ ? Use what you know about this function to find its equation. Verify that as  $x$  increases, the values of  $y$  get closer to  $y = 10$ .

$$y = a \cdot b^x + c$$

↑  
horizontal asymptote  
 $c = 10$

- d. Find the y-intercept of the function. What is the connection between the y-intercept and the asymptote?

SUMMARY:

Find an exponential equation through  $(3, 12.5)$  and  $(4, 11.25)$   
and has a horizontal asymptote  $y = 10$

Goal:  $y = a \cdot b^x + c$

$(3, 12.5)$

$$12.5 = a \cdot b^3 + 10$$

$$2.5 = a \cdot b^3$$

$$\frac{2.5}{b^3} = a$$

$$\frac{2.5}{(.5)^3} = a$$

$$\frac{2.5}{(\frac{1}{2})^3} = a$$

$$\frac{2.5}{\frac{1}{8}} = a$$

$$20 = a$$

$(4, 11.25)$

$$11.25 = a \cdot b^4 + 10$$

$$1.25 = a \cdot b^4$$

$$1.25 = \frac{2.5}{b^3} \cdot b^4$$

$$1.25 = 2.5b$$

$$\frac{1.25}{2.5} = b$$

$$0.5 = b$$

$$y = 20 \cdot 0.5^x + 10$$