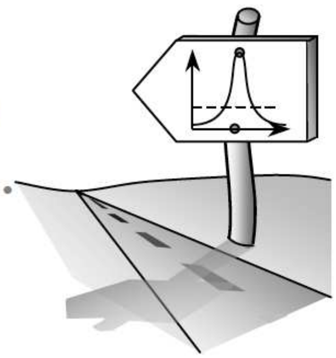
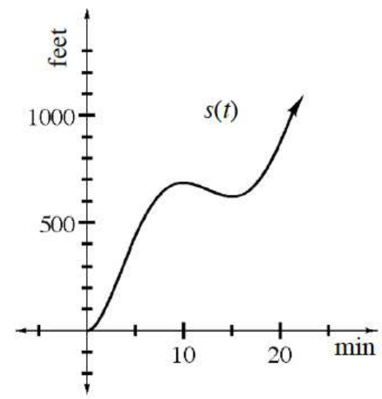
6.4.2 What does mean value really mean?

Mean Value Theorem



6-134. Is the velocity function needed in order to find average velocity? Examine this same situation from a different perspective. Lesson 6.4.1 Resource page



a. Using Steven's velocity function from Lesson 6.4.1, find a function $s(t)$ representing his distance from home during his 25-minute walk. The graph of $s(t)$ should match the graph at right.

b. According to $s(t)$, how far from school does Steven live? Determine his average velocity. Explain your method.

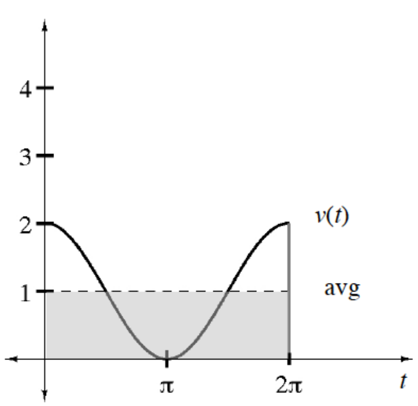
c. Locate Steven's distance graph on the Lesson 6.4.1 Resource page. Graph Steven's trip if he had traveled the average velocity during the entire trip. What does this graph look like? Label it $k(t)$.

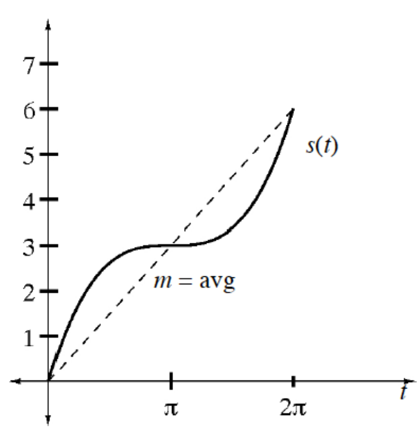
d. How many times during his trip was Steven actually traveling at the average velocity? Find these exact times.

e. Compare both graphs and determine how the average velocity is represented on each.

6-135. MAKING CONNECTIONS: MEAN VALUES

In problems 6-123 and 6-134, you found the same average velocity in two distinctly different ways. What is the connection? Study the graphs of velocity $v(t)$ and position $s(t)$ below and consider the relationship between distance and velocity. Notice the different representations of the average value. Then write down your observations.





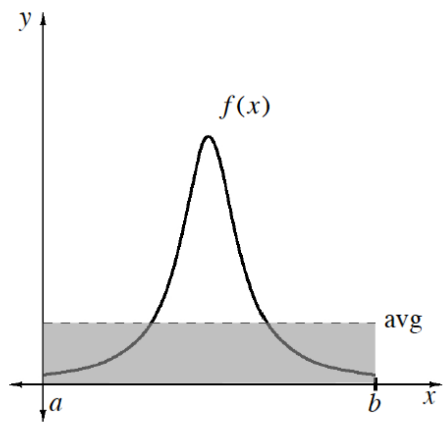
MATH NOTES

Average (Mean) Values

To find the **average (mean) value** of a set of items, we add up the items and divide by the number of items.

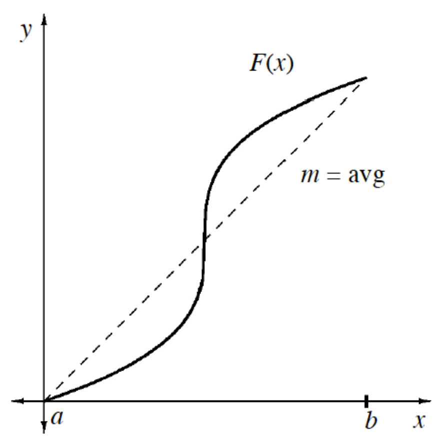
Integrals help us add over a continuous interval. Therefore, for any continuous function $f(x)$:

$$\frac{\int_a^b f(x)\,dx}{b-a} = \text{average value of } f(x) \text{ on } [a, b]$$



Since $\int_a^b f(x)\,dx = F(b) - F(a)$, we can also find the average value of any function $f(x)$ using its antiderivative $F(x)$. Its average slope gives the average rate of change of $F(x)$, which is the *same* as the average value of $f(x)$.

$$\frac{\int_a^b f(x)\,dx}{b-a} = \frac{F(b)-F(a)}{b-a} = \quad \text{avg. value of } F'(x)$$



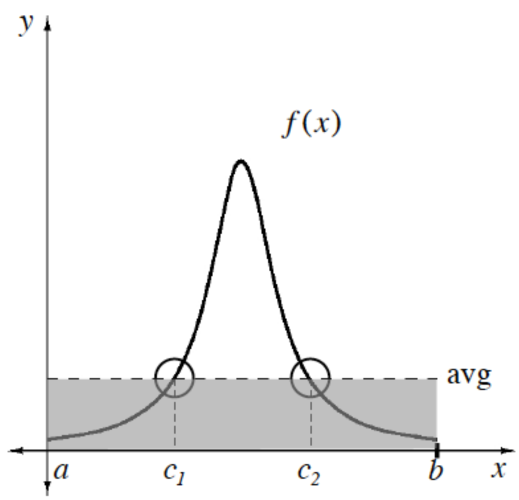
MATH NOTES

The Mean Value Theorem of $f(x)$

The **Mean Value Theorem *of f*(*x*)**(given ***f*(*x*)**).

If $f(x)$ is continuous on $[a, b]$, then there exists at least
one point c in (a, b) such that

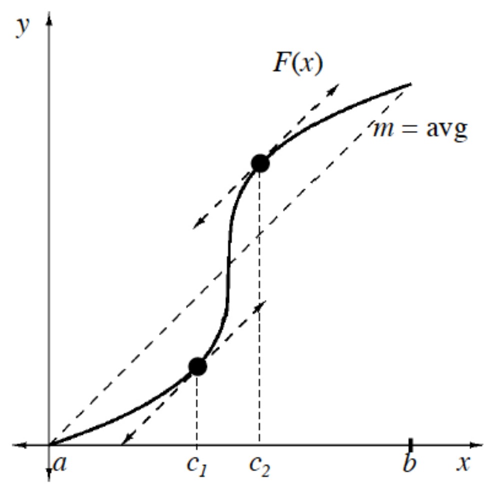
$$\frac{1}{b-a}\int_a^b f(x)dx = f(c).$$



The **Mean Value Theorem of *f*(*x*)**
(given ***F*(*x*)**)

If $F(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there exists at least one point c in (a, b) such that

$$F'(c) = \frac{F(b)-F(a)}{b-a} = f(c).$$



6-136. Let $f(x) = x^2$.

a. Demonstrate the Mean Value Theorem by finding a point c in $(-1, 2)$ such that $f(c)$ is the average value of x^2 over the interval.

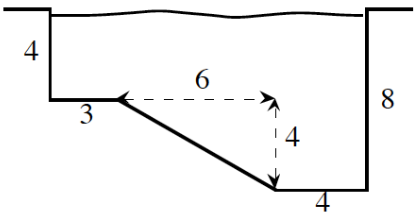
b. Sketch the graph of $f(x)$ over $[-1, 2]$ and see if your answer to part (a) seems plausible.

6-137. Let $f(x) = x^2$.

a. Demonstrate the Mean Value Theorem for Derivatives by finding a point c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$ (the slope of the graph of $f(x)$ when $x = c$ equals the average slope from beginning to end) when $a = -1$ and $b = 4$.

b. Sketch the graph of $f(x)$ over $[-1, 4]$. Does your answer to part (a) seem plausible?

6-138. In Cooperville, to get a permit for a swimming pool, the average depth of the pool must be less than 6 feet. The company installing your pool has designed a pool so that along the length, x, of the pool, the depth, $d(x)$, is:



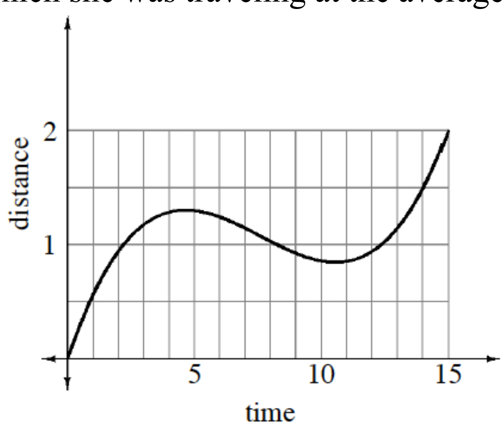
$$d(x)=\begin{cases} 4 & \text{for } 0 \le x < 3 \\ \frac{2}{3}x+2 & \text{for } 3 \le x < 9 \\ 8 & \text{for } 9 \le x \le 13 \end{cases}$$

Calculate the average (mean) depth of the swimming pool. Will the pool meet the permit guidelines?

6-139. During traffic it took Mitzi 15 minutes to travel two miles!

a. What was her average velocity?

b. Sketch the graph below. On the graph, locate the time(s) during her trip at which she was traveling at the average velocity. Explain your method.



c. Do you think that Mitzi must have a time at which she is traveling the average velocity? Why or why not?

Review & Preview

6-140. WHAT A CARD!

The value, V, of a Honus Wagner baseball card in mint condition appreciates at 12% per year. It was sold at auction in 1991 for \$461,000. If t= time in years since 1991, then $V = 461000(1.12)^t$. Homework Help ✎

a. What was its value in 2000?

b. Find the average value of the card over the period 1991 - 2000.

c. When was the card worth the value you found in part (b)?

6-141. Find and compare the average value of each of these functions over one complete cycle. Homework Help ✎

a. $f(t) = \sin t$

b. $g(t) = |\sin t|$

c. $k(t) = (\sin t)^2$

6-142. Without using your calculator, evaluate the following expressions. Homework Help ✎

a. $\frac{d}{dx}\left(\int_2^x \sqrt{9-x^3}\,dx\right)$

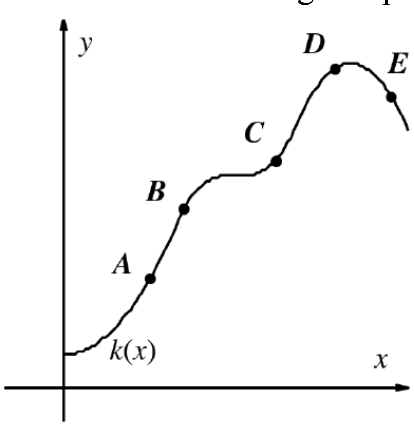
b. $\frac{d}{dt}(t^{-1}\sin t)$

c. $\int\left[\frac{d}{dx}\left(\frac{x+\tan x}{x-\sec x}+5\right)\right]dx$

d. $\int_0^1\left[\frac{1}{\sqrt{1-x^2}}+\sec^2 x\right]dx$

6-143. Humberto thinks the second derivative has something to do with how well a line will approximate a curve. He thinks that a line will better approximate a curve at $(a, f(a))$ if, for example, $f''(a) = 0.85$ than if $f''(a) = 5$. Is he correct? Why or why not? Use a graph to justify your answer. Homework Help ✎

6-144. While answering the questions below, refer to the graph of $k(x)$ below. Homework Help ✎



a. Between which pair of consecutive points is the average rate of change of $k(x)$ the greatest?

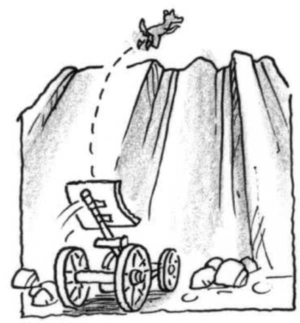
b. Assuming the axes are scaled equally, at which point is the instantaneous rate of change closest to 1?

c. Which is greater: the average rate of change between B and C or the instantaneous rate of change at C?

d. Which is greater: the average rate of change between C and D or the average rate of change between C and E?

6-145. What is the best linear approximation of the general function $y = \sin(ax)$ near $x = 0$? Homework Help ✎

6-146. Wile E. Coyote is so desperate to catch the Roadrunner that he built himself a catapult to use on those darn vertical cliffs he is always running into. He figures he will need it to shoot him 500 meters straight up. How fast must the catapult propel him upwards to reach that height? (Use -5 m/sec^2 for the acceleration due to gravity. As you know, Cartoonland has a different gravitational constant, allowing cartoon characters to fall from great heights without being injured. Do not try this at home.) Homework Help ✎



6-147. The Business Section of the Metropolis Express lists Megabux Bank as paying 4.95% compounded daily on a $10,000 deposit, for a "yield" (or overall gain) of 5.07%. Homework Help ✎

a. Compute how much the initial $10,000 deposit will be worth one year after it is deposited at Megabux.

b. Confirm the 5.07% "yield."

c. When will the initial $10,000 deposit be worth $15,000?

6-148. The last remaining parent graph without a derivative is the circle $x^2 + y^2 = 1$. Find $\frac{dy}{dx}$ for this graph. Homework Help ✎

6-149. Recall that $e^{\ln x} = x$, use the derivative of the inverse to find $\frac{d}{dx}(\ln x)$. Homework Help ✎