6.4.3 How is average value shown in both distance and velocity graphs?

Mean Value Theorem: Applications



6-150. The velocity of Kamilah's new car during a six-hour trip is $v(t) = -3t^2 + t^2$ 17t + 6, where v(t) is measured in miles per hour.

- a. When is the car at rest?
- b. Sketch Kamilah's velocity over time and find her average velocity during her six-hour trip. Let v_{avg} represent the average velocity.



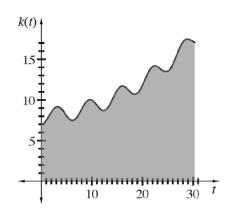
c. Graph Kamilah's velocity and the equation $y = v_{avg}$ on the same set of axes. Then compare the areas represented by the integrals below. What do you notice?

$$\int_0^6 v(t)dt \text{ and } \int_0^6 v_{\text{avg}} dt$$

- d. Find s(t), the distance Kamilah's car travels over time. Assume s(0) = 0. Then use s(t) to find the average velocity of the sports car. Do your results match part (b)?
- e. Determine when Kamilah's velocity equals her average velocity. Justify your answer.

6-151. ELECTRIC COMPANY

Kira has been hired by the billing department of her local electric company. Her task is to track electricity usage over the course of the month and provide her clients with statements that include the average monthly use of electricity. This way, customers can predict future bills and can determine if their monthly usage is typical. Since the rate electricity at which is consumed fluctuates throughout the day and varies from day-to-day, Kira uses a function to calculate the number of kilowatts being used at time t. (Kilowatts are a unit of electrical power. They measure the *rate* of electrical energy use.) For one of her customers, the electricity usage in kilowatts for a 30-day month is given by:



$$k(t) = 8 - \cos t + 0.01t^2 + 0.01t$$
, where t is in days.

a. Using k(t), find the average power used in kilowatts for this month. Be prepared to explain your method to the class.

b. On which days during the 30-day month did Kira's average kilowatt usage equal her actual kilowatt usage. Justify your answer.

MATH NOTES



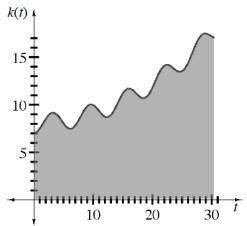
Rolle's Theorem

If f is continuous on a closed interval [a,b] and differentiable on the open interval (a,b) and if f(a) = f(b), then f'(c) = 0 for at least one number c in the open interval (a,b).

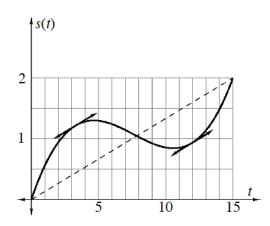
6-152. Use the Mean Value Theorem to explain why Rolle's Theorem must be true. Be sure to include a diagram in your explanation.

6-153. Demonstrate Rolle's Theorem using the function $f(x) = -x^4 + 2x^3 - 2x + 1$. Begin by finding the points a and b for which f(x) = 0.

6-154. Explain why there must be at least one moment during the month when a customer's kilowatt usage is equal to his or her average kilowatt usage.



6-155. Remember Mitzi? The graph below below shows the two "average" tangents that represent the times Mitzi is traveling at the average velocity during her trip from problem 6-139. The slope of the dotted line represents her average velocity, since it represents traveling 2 miles in 15 minutes.



- a. Explain what the tangents represent.
- b. The equation for Mitzi's travel is given below. Compute the times at which Mitzi was traveling at the average velocity.

$$s(t) = 0.004426t^3 - 0.10113406t^2 + 0.654424523t$$

- **6-156.** Show that f(x) must be differentiable in order to apply the Mean Value Theorem by providing a counterexample. Be sure to specify a particular function and a particular interval that shows that if f(x) is *not* differentiable over (a, b), then the Mean Value Theorem does not necessarily hold.
- **6-157.** Find the average rate of change of the function $\int_{1}^{\infty} \frac{1}{x^2} dx$ over the interval [1, 3].



6-158. Use the relationship between differentiation and integration to simplify the following problems. Homework Help

a.
$$\frac{d}{dx}(\cos(3x^2))$$

b.
$$\int 6x \sin(3x^2) dx$$

c.
$$\frac{d}{dx}(\sqrt{3x^9-5x})$$

d.
$$\int \frac{27x^8-5}{\sqrt{3x^9-5x}} dx$$

- **6-159.** If $\frac{dy}{dx} = 3x$, find y as a function of x. Homework Help \(\)
- **6-160.** The function $k(t) = 8 \cos t + 0.01t^2 + 0.01t$ helped Kira determine the *rate* at which electricity was being used on any day t of the month. However, to manage total electrical production, Kira needs to determine a function that will accumulate the energy used over the course of the month. That is, she needs a function K(t) that will determine the total energy used from the beginning of the month to time t.

Homework Help 🔪

- a. Find a function K(t) that will accumulate the power used over time t. Be sure that K(0) = 0. What is its relationship to K(t)?
- b. Use K(t) to find the total electrical energy used during this month.
- c. Sketch a graph of K(t) for this 30-day period.
- d. Calculate the average power used in kilowatts for this month. Did your answer match that in problem 6-151? How is this rate represented geometrically in the graph?
- **6-161.** Find the average value of g(t) defined below over the given interval. You may solve analytically or use your graphing calculator to evaluate. Homework Help $\$

a.
$$g(t) = 2t^2 - 3$$
 over $[0, 4]$

b.
$$g(t) = 3e^t + \ln(t+1)$$
 over $[0, 1]$

- **6-162.** Find the equation of the line tangent to the graph of $e^{xy} + \ln y = 1$ at the point (0, 1). Homework Help
- **6-163.** If g(x) is a differentiable function such that g(x) < 0 for all real numbers x and if $f'(x) = (x^2 9)g(x)$, determine if f(3) and f(-3) are relative minimums or maximums. Justify your answer. Help