Chapter 6 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

1. TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, a list of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

Next consider the Standards for Mathematical Practice that follow " ③ PORTFOLIO". What Mathematical Practices did you use in this chapter? When did you use them? Give specific examples.

How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Learning Log Entries

- Lesson 6.1.1 – Plotting Points in *xyz*-Space

- Lesson 6.1.4 – Systems of Three Equations with Three Variables

- Lesson 6.1.5 – Finding the Equation of a Parabola Given Three Points

- Lesson 6.2.2 – Logarithm Properties

Math Notes

- Lesson 6.1.2 – Locating Points in Three Dimensions

- Lesson 6.1.4 – Graphing Planes in Three Dimensions

- Lesson 6.2.2 – Logarithm Properties

2. MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

3-D coordinate system	**asymptote**
isometric dot paper	**logarithm**
ordered triple	**plane**
Power Property of Logs	**Product Property of Logs**
Quotient Property of Logs	**solution**

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection. A word can be connected to any other word as long as you can justify the connection.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

3. PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

This section gives you an opportunity to show growth in your understanding of key mathematical ideas over time as you complete this course.

Part 1: You may have done this problem before. You have grown mathematically since then. Explain everything that you know about $y = x^2 - 4$ and $y = \sqrt{x + 4}$. Be sure to include everything you have learned since the last time you did this problem.

Part 2: Compare your responses to this "Growth Over Time" problem. Consider each of the following questions as you write an evaluation of your mathematical growth. If you only did the problem twice, then answer accordingly.

- What new concepts did you include the second time you did the problem? In what ways was your response better than your first attempt?

- How was your final version different from the first two? What new ideas did you include?

- Did you omit anything in the final version that you used in one of the earlier versions? Why did you omit it?

- Is there anything you want to add to your current version? If so, add it and keep this version for future reference.

- Rate your three attempts by making three bars like the ones below and shading each bar (left to right) to represent how much you knew on each attempt.

First Attempt:
Second
Final Attempt:

Alternatively, your teacher may ask you to showcase your ability to use logarithms to solve a problem involving an exponential function. Copy your work from "The Case of the Cooling Corpse," problem 6-137, and enhance it if needed.

Next consider the Standards for Mathematical Practice that follow. What Mathematical Practices did you use in this chapter? When did you use them? Give specific examples.

Your teacher may give you the Chapter 6 Closure Resource Page: Multiple Representations of Logarithmic Functions Graphic Organizer page to work on.

BECOMING MATHEMATICALLY PROFICIENT
The Common Core State Standards For Mathematical Practice

This book focuses on helping you use some very specific Mathematical Practices. The Mathematical Practices describe ways in which mathematically proficient students engage with mathematics everyday.

Making sense of problems and persevering in solving them:

Making sense of problems and persevering in solving them means that you can solve problems that are full of different kinds of mathematics. These types of problems are not routine, simple, or typical. Instead, they combine lots of math ideas and everyday situations. You have to stick with challenging problems, try different strategies, use multiple representations, and use a different method to check your results.

Reason abstractly and quantitatively:

Throughout this course, everyday situations are used to introduce you to new math ideas. Seeing mathematical ideas within a context helps you make sense of the ideas. Once you learn about a math idea in a practical way, you can "**reason abstractly**" by thinking about the concept more generally, representing it with symbols, and manipulating the symbols. **Reasoning quantitatively** is using numbers and symbols to represent an everyday situation, taking into account the units involved, and considering the meaning of the quantities as you compute them.

Construct viable arguments and critique the reasoning of others:

To **construct a viable argument** is to present your solution steps in a logical sequence and to justify your steps with conclusions, relying on number sense, facts and definitions, and previously established results. You communicate clearly, consider the real-life context, and provide clarification when others ask. In this course, you regularly share information, opinions, and expertise with your study team. You **critique the reasoning of others** when you analyze the approach of others, build on each other's ideas, compare the effectiveness of two strategies, and decide what makes sense and under what conditions.

Model with mathematics:

When you **model with mathematics**, you take a complex situation and use mathematics to represent it, often by making assumptions and approximations to simplify the situation. Modeling allows you to analyze and describe the situation and to make predictions. For example, you model when you use multiple representations, including equations, tables, graphs, or diagrams to describe a situation. In situations involving the variability of data, you model when you describe the data with equations. Although a model may not be perfect, it can still be very useful for describing data and making predictions. When you interpret the results, you may need to go back and improve your model by revising your assumptions and approximations.

Use appropriate tools strategically:

To **use appropriate tools strategically** means that you analyze the task and decide which tools may help you model the situation or find a solution. Some of the tools available to you include diagrams, graph paper, calculators, computer software, databases, and websites. You understand the limitations of various tools. A result can be check or estimated by strategically choosing a different tool.

Attend to precision:

To **attend to precision** means that when solving problems, you need to pay close attention to the details. For example, you need to be aware of the units, or how many digits your answer requires, or how to choose a scale and label your graph. You may need to convert the units to be consistent. At times, you need to go back and check whether a numerical solution makes sense in the context of the problem.

You need to **attend to precision** when you communicate your ideas to others. Using the appropriate vocabulary and mathematical language can help make your ideas and reasoning more understandable to others.

Look for and make use of structure:

To **looking for and making use of structure** is a guiding principal of this course. When you are involved in analyzing the structure and in the actual development of mathematical concepts, you gain a deeper, more conceptual understanding than when you are simply told what the structure is and how to do problems. You often use this practice to bring closure to an investigation.

There are many concepts that you learn by looking at the underlying structure of a mathematical idea and thinking about how it connects to other ideas you have already learned. For example, you understand the underlying structure of an equation such as $y = a(x - h)^2 + b$ which allows you to graph it without a table.

Look for and express regularity in repeated reasoning:

To **look for and express regularity in repeated reasoning** means that when you are investigating a new mathematical concept, you notice if calculations are repeated in a pattern. Then you look for a way to generalize the method for use in other situations, or you look for shortcuts. For example, the pattern of growth you notice in a geometric sequence results in being able to write a general exponential equation that highlights the growth and starting point.

4. WHAT HAVE I LEARNED

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 6-148. Graph in three dimensions.

  a. (2, 3, 1)

b. $(-2, 3, 0)$

c. $2x + y - z = 6$

CL 6-149. Determine the point of intersection of the three planes.

a. $x + y + z = 3$
$2x - y + 2z = 6$
$3x + 2y - z = 13$

b. $x + y + 4z = 5$
$-2x + 2z = 3$
$3x + y - 2z = 0$

CL 6-150. The parabola $y = ax^2 + bx + c$ passes through the points $(2, 3)$, $(-1, 6)$, and $(0, 3)$. Determine:

a. The equation of the parabola.

b. The vertex of the parabola.

c. The x-intercepts of the parabola.

CL 6-151. Solve each equation to the nearest thousandth (0.001).

a. $2^x = 17$

b. $5x^3 = 75$

c. $5(3^{x+1}) = 85$

d. $\log_3(x + 1) = -2$

CL 6-152. A gallon of propane costs $3.59. Inflation has steadily increased 4% per year.

a. What did a gallon of propane cost ten years ago?

b. If this trend continues, how much longer will it be until it costs $10?

CL 6-153. Find the inverse of this equation: $y = 2 + \sqrt{2x - 4}$

CL 6-154. Use your Parent Graph Toolkit or make a table to graph $y = \log_2(x)$.

CL 6-155. Use your answer to the previous problem to graph $y = 1 + \log_2(x - 3)$.
State the equation of the new asymptote and the new x-intercept.

6-156. Solve each of the following equations. Be sure to check your answers.

a. $3|2x - 5| - 8 = -5$

b. $\sqrt{3x^2 + 11x} = 2$

6-157. Graph each of the following systems of inequalities.

a. $y \geq 3(x - 2)^2 - 4$
$y > -2|x - 1| + 3$

b. $y > (x - 1)^2 - 5$
$y > 3x - 5$
$y \leq \frac{1}{2}(x - 1)^2 + 1$

CL 6-158. Consider the function $f(x) = \sqrt{x+3}$
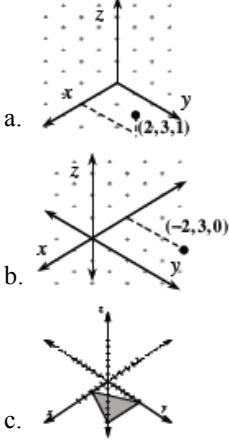
    a. What are the domain and range of $f(x)$?

    b. If $g(x) = x - 10$, what is $f(g(x))$?

    c. What are the domain and range of $f(g(x))$?

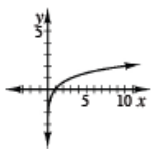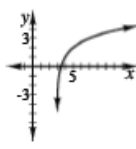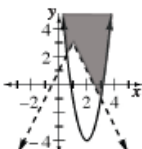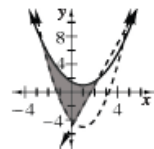   d. Is $f(g(x)) = g(f(x))$ Justify why or why not.

CL 6-159. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in previous math classes? Sort the problems into three groups: the ones you are confident you can do, the ones you need more practice with, and the ones you need further help to understand.

Answers and Support for Closure Activity #4
What Have I Learned?

Note: MN = Math Note, LL = Learning Log

Problem	Solutions	Need Help?	More Practice
CL 6-148.	a. b. c.	Lessons 6.1.1 and6.1.2 MN: 6.1.2 and 6.1.4 LL: 6.1.1	Problems 6-6 and 6-19
CL 6-149.	a. $(4, 0, -1)$ b. $(-1, 4, 0.5)$	Lesson 6.1.4 LL: 6.1.4	Problems 6-48, 6-51, 6-71, 6-80, and 6-103
CL 6-150	a. $y = x^2 - 2x + 3$ b. $(1, 2)$ c. No x-intercepts	Lesson 6.1.5 LL: 6.1.5	Problems 6-64, 6-67, 6-68, 6-69, 6-72, 6-81, and6-131
CL 6-151.	a. 4.087 b. 2.466 c. 1.579 d. −0.889	Lessons 6.2.1 and6.2.2	Problems 6-11, 6-94, 6-113, 6-133, and 6-140
CL 6-152.	a. $2.43 b. 26.1 years	Lessons A.3.2 andB.2.3	Problems 6-15, 6-52, 6-117, and 6-138
CL 6-153.	$y = \frac{(x-2)^2}{2} + 2$	Lessons 5.1.1 and5.1.3 LL: 5.1.1	Problems 6-102, 6-120, 6-132, and 6-136

CL 6-154.		Lessons 5.2.2 and 5.2.3 LL: 5.2.3	Problems 5-68, 5-69, and 5-94
CL 6-155.	 Asymptote $x = 3$ Intercept (3.5, 0)	Lesson 5.2.4	Problems 5-94 and 5-118
CL 6-156.	a. $x = 2, 3$ b. $x = \frac{1}{3}, -4$	Lessons 4.1.2 and 4.1.3 LL: 4.1.1 and 4.1.4	Problems 4-22, 4-23, 6-40, 6-53, and 6-76
CL 6-157.	 a. b.	Lesson 4.2.1 MN: 4.2.3 and 4.2.4 LL: 4.2.1	Problems 5-13, 5-67, 6-33, 6-58, and 6-147
CL 6-158.	a. domain: $x \geq -3$, range: $y \geq 0$ b. $f(g(x)) = \sqrt{x-7}$ c. domain: $x \geq 7$, range: $y \geq 0$ d. no, $g(f(x)) = \sqrt{x+3} - 10$ and $\sqrt{x+3} - 10 \neq \sqrt{x-7}$	Lesson 5.1.3 MN: 5.1.3 LL: 5.2.5	Problems 6-87, 6-132, and 6-141