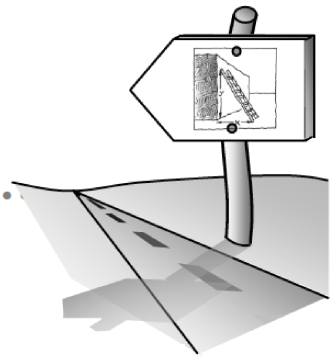
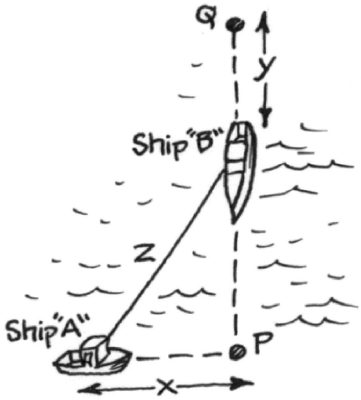
7.1.2 How do I model rates of change?



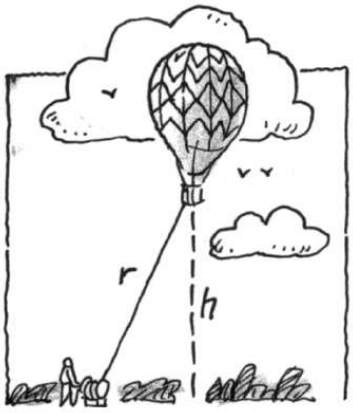
Related Rates Applications: The Pythagorean Theorem

7-13. For the following scenarios, write related rate statements.



a. Ships A and B are attached by a long cable of length z. As ship B moves toward point Q, Ship A heads toward point P. Describe the related rates $\frac{dy}{dt}$ and $\frac{dx}{dt}$. Are they positive or negative? If $\frac{dy}{dt}$ is constant, how does $\frac{dx}{dt}$ change?

b. A hot air balloon rises vertically from a platform as a rope of length r is slowly released. What is the relationship between the rates $\frac{dr}{dt}$ and $\frac{dh}{dt}$? Are they positive or negative? If $\frac{dr}{dt}$ is constant, how does $\frac{dh}{dt}$ change?



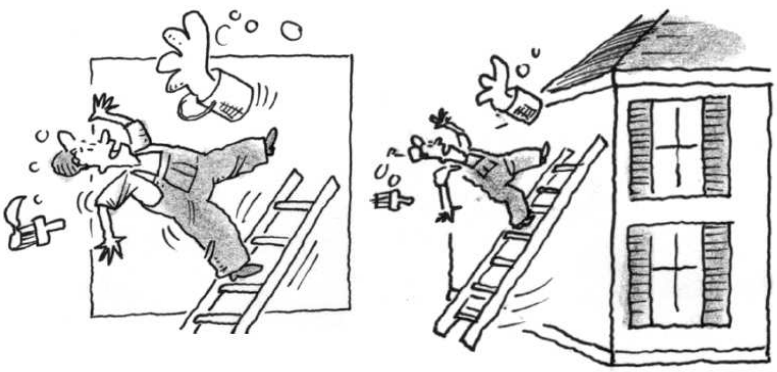
7-14. PAINFUL PAINTING, Part One

While painting his house one day, Mr. Cabana had a terrible accident! The bottom of the 20-foot ladder he was standing on started to slide away from the wall!

a. Draw a diagram of this situation. What calculus tools can be applied to this situation?

b. Explain why the rate the ladder is sliding down the wall must be related to the rate that the foot of the ladder moving along the ground.

7-15. PAINFUL PAINTING, Part Two



Remember from problem 7-14 that Mr. Cabana was standing on top of a 20-ft ladder when the base started to slide away from the wall. If the base slides at 1.5 ft/sec we want to find the rate at which he is falling. Let h be the height of the ladder at a given time t.

a. List all the given facts about this scenario. Pay close attention to notation and units.

b. We want to find the rate at which Mr. Cabana is falling. Translate this question into symbols. What will you need to solve for? What are the units?

c. Does Mr. Cabana fall at the same rate throughout, or is his rate changing?

d. Since we need to find $\frac{dh}{dt}$, we first need an equation that involves h. Find this equation.

e. Rewrite your equation to find $\frac{dh}{dt}$. Then use it to find $\frac{dh}{dt}$ when the base of the ladder is 10 ft from the building.

f. Explain why $\frac{dh}{dt}$ should be negative.

7-16. While working with a loop of yarn, Sophia caused the length l of the rectangle to *increase* at a rate of $3\ \frac{\text{cm}}{\text{sec}}$.

Help Sophia find the rates below when $l = 15$ cm and $w = 5$ cm. Be sure to include a diagram labeled with appropriate units, and tell whether each quantity is increasing or decreasing.

a. Find the rate of change of the perimeter of the rectangle.

b. Find $\frac{dw}{dt}$.

c. Find the rate of change of the area of the rectangle.

d. Find the rate of change of the length of a diagonal of the rectangle.

7-17. Compute the following derivatives. Homework Help

a. $\frac{d}{dx}\left[\sin^3(x)\right]$

b. $\frac{d}{dk}\left[\sqrt[3]{7+4k-2k^2}\right]$

c. $\frac{d}{dx}\left[\pi^x x^\pi\right]$

7-18. Hustling Harry hastily got the following answer to the indefinite integral below. Verify that he is incorrect, and explain what he did wrong. Homework Help ✎

$$\int 2x\cos(x^2)dx = x^2\sin\left(\tfrac{1}{3}x^3\right)+C$$

7-19. It so happens that Hustling Harry's grade in the class at any week t during the semester is calculated by $g(t) = 46\cos\frac{t}{10} + 10\sin\frac{t}{2} + 40$. At what point during the 18-week semester was Harry's grade equal to his average grade for the semester? Homework Help ✎

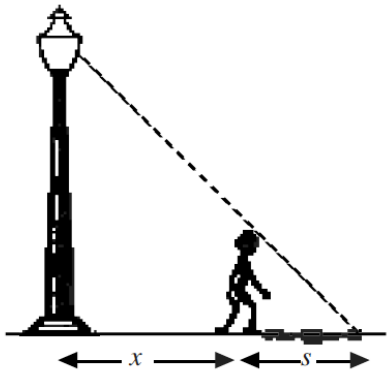
7-20. A point travels along the x-axis so that at time t, where $0 \le t \le 5$, its position is $s(t) = t^3 - 5t^2 + 4t$. Homework Help ✎

a. What does $s(t)$ represent?

b. Where is the particle located when $t = 5$?

c. Find the velocity of the particle when its acceleration is zero.

7-21. Patrick, who is 6 feet tall, is walking way from a 10-foot tall lamppost. If he casts a shadow that is 12.5 ft long, how far from the lamppost is he? Homework Help ✎



7-22. Find y if $\frac{dy}{dx} = 3x^{-1/4} - \frac{7}{2}\sqrt{x}$. Homework Help ✎

7-23. THE WEDDING CAKE, Part One

Kiki is ordering a wedding cake for her upcoming reception. She is considering a four-layer cake, where the diameters are 16, 12, 8, and 4 inches. Each layer will be 4 inches tall. Homework Help ✎

a. Write a Riemann sum to approximate the volume of her cake.

b. The bakery has informed her that she should intend to have approximately 7 in^3 of cake per guest at the reception. She expects 200 guests at her wedding. Will she have enough cake?

7-24. Explain how the first derivative can determine a local minimum or a maximum. Be sure to distinguish between the two. (Note: What if the first derivative is undefined?) Homework Help ✎