Lesson 7.1.2

7-13. See below:

a. Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are negative. If $\frac{dy}{dt}$ is constant, $\frac{dx}{dt}$ will decrease more and more.

b. Both $\frac{dr}{dt}$ and $\frac{dh}{dt}$ are positive. If $\frac{dr}{dt}$ is constant, $\frac{dh}{dt}$ is positive but decreases to approach $\frac{dr}{dt}$.

7-14. See below:

a. Answers may vary, but students should see that this situation involves rates - both the rate at which the ladder is slipping and the rate at which Mr. Cabana is falling.

b. Because the ladder is of fixed length, when the ladder moves down the wall, the bottom is forced to move accordingly.

7-15. See below:

a. The only constants are the 20-ft length of the ladder and $\frac{dx}{dt} = 1.5 \frac{\text{ft}}{\text{sec}}$. Encourage students to label distances and associated rates in their diagrams. In this case, the distance along the ground should be labeled x, and should also show that $\frac{dx}{dt} = 1.5$.

b. The word "rate" in the question tells us we are looking for a derivative. Since the units for the answer would be $\frac{\text{ft}}{\text{sec}}$, and the "feet" refers to height, we are looking for $-\frac{dh}{dt}$.

c. His rate is increasing despite the fact that $\frac{dx}{dt}$ is constant.

d. $h^2 + x^2 = 20^2$ is the most convenient. Trigonometric equations are also possible, but we are not given the rate of change of the angles.

e. Differentiating implicitly with respect to t gives $2h \frac{dh}{dt} + 2x \frac{dx}{dt} = 0$, Substitute $\frac{dx}{dt} = 1.5 \frac{\text{ft}}{\text{sec}}$, $x = 10$, $h \approx 17.321$ ft, and $\frac{dh}{dt} \approx -0.866 \frac{\text{ft}}{\text{sec}}$.

f. Because Mr. Cabana is falling!

7-16. See below:

a. 0

b. –3 cm/sec

c. $A = lw$, so $\frac{dA}{dt} = l\,\frac{dw}{dt} + \frac{dl}{dt} = -30\ \frac{\text{cm}^2}{\text{sec}}$

d. ≈ 1.897 cm/sec

7-17. See below:

a. $3 \sin^2 x \cos x$

b. $\frac{1}{3}(7 + 4k - 2k^2)^{-2/3}(4 - 4k)$

c. $(\ln \pi)x^{\pi}\,\pi^x + \pi^{x+1}\,x^{\pi - 1}$

7-18. He ignored the Product Rule.

7-19. Avg. grade ≈ 67.011, which was Harry's grade at $t \approx 7.63$ in the middle of the eighth week.

7-20. See below:

a. Position on the x-axis.

b. At $x = 20$.

c. $-\frac{13}{3}$

7-21. $8\frac{1}{3}$ ft

7-22. $y = 4x^{3/4} - \frac{7}{3}x^{3/2} + C$

7-23. See below:

a. $4\pi\sum_{i=0}^{3}(8-2i)^2$

b. Yes: $V = 4\pi(8^2 + 6^2 + 4^2 + 2^2) = 1507.964$ in^3, leaving ≈ 7.40 in^3 per guest.

7-24. Students should mention that the locations of local max and mins can be determined when $f'(x) = 0$ or $f'(x)$ is undefined. Students should also discuss strategies to distinguish between a local max and a local min.