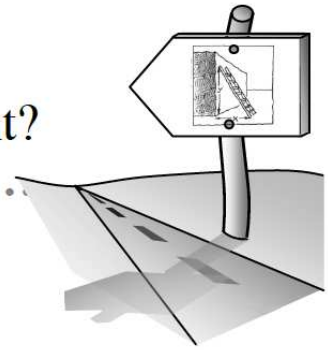
7.1.4 Why are good diagrams so important?



Related Rates Application: Choosing the Best Formula

MATH NOTES

Solving Related Rate Problems

The basic process for solving any related rates problem is:

1. Use a diagram and geometry to find a relationship between the changing quantities. For example, is there an appropriate area or volume formula that you could apply? Is there a right triangle for which the Pythagorean Theorem could be applied? Are there similar triangles?

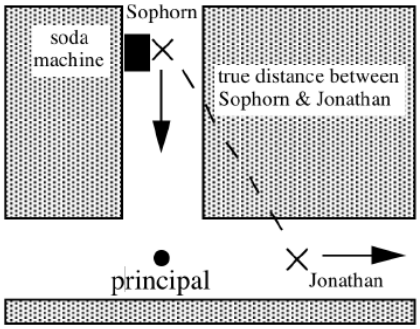
2. Use implicit differentiation to find a rate relationship between the terms. Do not forget to differentiate with respect to time (if time is important)!

3. Substitute quantities and rates to solve for the desired rate. Be sure to make any decreasing rates negative.

7-37. Happy the Clown has balloons that are perfect spheres. Happy fills the balloons with helium at a rate of 20 in^3 per second. The recommended volume for the balloons is 500 in^3. Happy wants to know how fast the diameter is changing when the volume is at 500 in^3. Happy is now sad because he knows his answer shown below does not make sense. Find and fix his error so that Happy will be happy again.

$$V = \tfrac{4}{3}\pi r^3$$
$$500 = \tfrac{4}{3}\pi r^3$$
$$0 = 4\pi r^2 \tfrac{dr}{dt}$$
$$0 = \tfrac{dr}{dt}$$

7-38. LATE TO CLASS



Sophorn and Jonathan are late to class*again*, but this time the principal caught them. Sophorn, who has a pass, started by the soda machine and hustled toward the principal at $4\,\frac{\text{ft}}{\text{sec}}$. At that same instant, Jonathan, who does not have a pass escaped from the principal, running away at a speed of $6\,\frac{\text{ft}}{\text{sec}}$.

a. If "h" represents the true distance between Sophorn and Jonathan, find an expression for $\frac{dh}{dt}$, the rate of change of the distance over time.

b. The soda machine, where Sophorn starts, is 15 feet from the principal. Find the rate of change of the distance between Sophorn and Jonathan when $t = 0$, 1, and 3 seconds. Note that at $t = 0$, Jonathan is at the same position as the principal.

c. Would the rates you found in part (b) change if Sophorn and Jonathan each were traveling at different rates than those given above? Why or why not?

7-39. A punch glass is in the shape of a hemisphere with radius of 5 cm. If punch is being poured into the glass so that the change in height of the punch is 1.5 cm/sec, find the rate the exposed area of the punch is changing when the height of the punch is 2 cm.

Review & Preview

7-40. As clumsy Kenny poured his cold coffee on the floor, he noticed that the growing puddle was circular. Let A be the area ofacircle with radius r at time t. Write an equation that relates $\frac{dA}{dt}$ to $\frac{dr}{dt}$. Homework Help

7-41. Let S be the surface area of an inflatable cube with sides of length x at time t. Write an equation that relates $\frac{dS}{dt}$ to $\frac{dx}{dt}$. Homework Help

7-42. Without a calculator, find the minimum value of $f(x) = 2x^3 - 15x^2 + 24x + 19$ for $\{x: 0 \le x \le 5\}$. Use the second derivative to justify that your value is a minimum. Homework Help

7-43. Two roads intersect at right angles. Elizabeth, who is driving east, leaves the intersection traveling at a speed of 40 mph. Two hours later, Nairi leaves the same intersection at a speed of 30 mph heading north. How fast is the distance between the cars changing five hours after the eastbound car leaves? Homework Help

7-44. Find the equation of the line(s) tangent to $y = x^2 + 2x + 4$ that pass through the origin. 7-44 HW eTool (Desmos). Homework Help

7-45. Kimberly and Varag are in a bicycle race. Homework Help

a. If Kimberly's velocity in miles per hour during the race is $v(t) = 2.3^t \sin\left(\frac{t}{2}\right)$, find the bicyclist's average velocity during $0 \le t \le 6$. Describe your method.

b. If Varag's distance from the starting line during the race is $s(t) = 20t + 2\sin(t)$, find his average velocity during $0 \le t \le 6$. Describe your method.

c. When is each bicyclist traveling at his/her average velocity?

7-46. Consider the curve formed by the equation $xy^2 - x^3y = 12$. Homework Help

a. Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

b. Find the x-coordinate of each point on the curve where the tangent line is vertical.