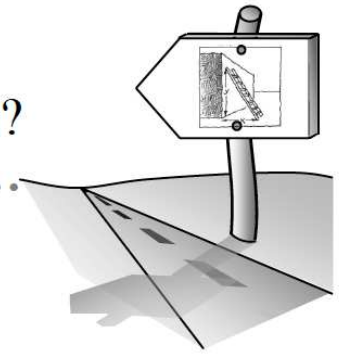
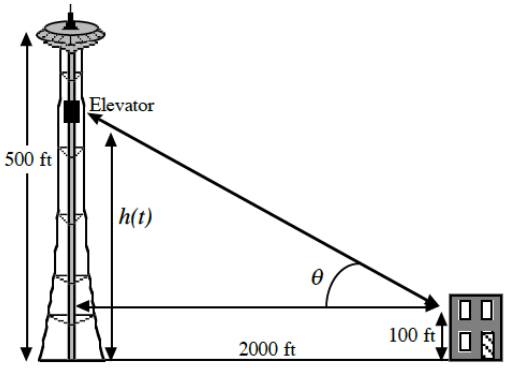
7.1.5 How do I measure a spinning object?



Related Rates Application: Trigonometry

7-47. STEEPNESS IN SEATTLE, Part One



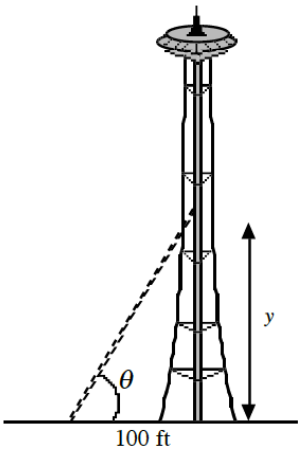
Seattle's Space Needle officially opened on the first day of the Seattle World's Fair, April 21, 1962. It features a flying saucer-like revolving restaurant 500 feet above the ground. The restaurant is accessible by stairs (832 steps), but most people take an elevator. Suppose you are across the street having coffee, watching the elevator descend. Your window is 100 feet above the ground and 2000 feet away from the Space Needle. The elevator descends at a constant speed of $14\,\frac{\text{ft}}{\text{sec}}$. Let θ be the angle of elevation of your line of sight to the elevator.

a. What is θ when the elevator is at the revolving restaurant? What is θ ten seconds later? Write an equation relating θ and $h(t)$.

b. Find an equation for the rate of change of θ with respect to t. Simplify your equation.

c. At what height will the elevator be when it appears to be moving fastest? What is θ at this height?

7-48. STEEPNESS IN SEATTLE, Part Two



When Liga visited Seattle, she tested the power of her laser pointer by shining it onto the Space Needle.

a. Liga is standing 100 ft away from the Space Needle. If y is the vertical distance from the ground to the laser's red dot, at what rate is y changing with respect to angle θ when $\theta = 1$ radian? Be sure to include the appropriate units in your answer.

b. At what rate does y change with respect to angle θ when Liga's laser hits the very top of the Space Needle, 605 ft?

c. Suppose Liga shines her laser at the elevator, which moves at $14\,\frac{\text{ft}}{\text{sec}}$. At what rate does θ change (with respect to time) when the elevator reaches the restaurant, 500 feet above ground? Remember the units!

Review & Preview

7-49. Find $\frac{dy}{dx}$ for each of the following functions. Homework Help

a. $y = 2^{3x^2 - x} + 6x^{-0.4}$

b. $y = 6x \ln(-3y) - x^{6^2}$

c. $y = \frac{3}{7}\cos(11^{0.2x})$

7-50. Integrate. Homework Help

a. $\int 10^{\pi - 1}\, dx$

b. $\int (9^{t-1})\, dt$

c. $\int \cos(4m - 3)\, dm$

7-51. An alien uses mysterious powers to make crop circles in a Nebraska wheat field. If the radius of a crop circle increases at the rate of 6 ft/min, find the rate at which the circle's area is increasing when the radius is 150 feet. Homework Help

7-52. As Khalid inflates a spherical balloon, Kareem wonders about its different rates. He knows that the rate at which Khalid blows is equal to the rate at which the volume changes $\left(\frac{dV}{dt}\right)$. As the balloon inflates, other aspects are changing as well, such as the radius and the surface area. Homework Help

a. If $\frac{dV}{dt} = 10\,\frac{\text{cm}^3}{\text{sec}}$, find the rate of change of the radius, $\frac{dr}{dt}$, when $r = 3$ cm.

b. If $\frac{dV}{dt} = 12\,\frac{\text{cm}^3}{\text{sec}}$, find the rate of change of the surface area, $\frac{dA}{dt}$, when $r = 5$ cm.

c. Describe what happens to the balloon when $\frac{dV}{dt}$ is negative.

7-53. Use the first and second derivatives to determine the following locations for $f(x) = xe^x$. Homework Help

a. Relative minima and maxima.

b. Intervals at which $f(x)$ is increasing and decreasing.

c. Inflection points.

d. Intervals at which $f(x)$ is concave up and concave down.

7-54. Find the average velocity of the traveling point described in problem 7-20. At what time(s) during the interval was the point traveling at this average velocity? Homework Help

7-55. Let $f'(x) = \frac{\ln|x-2|}{3}$ and $f(-3) = \frac{7}{3}$. Homework Help

a. Estimate $f(-3.1)$.

b. Find $f''(x)$.

c. Use concavity to determine if your answer to part (a) is an under or over estimate of the actual value of $f(x)$. Justify your answer.

7-56. Multiple Choice: Evaluate the expression $\int_0^{400}(5^x - 2^x)dx + \int_1^{400}(2^x - 5^x)dx$. Homework Help

a. 0.000

b. ≈ 1.043

c. ≈ 0.221

d. ≈ 1.820

e. $+\infty$