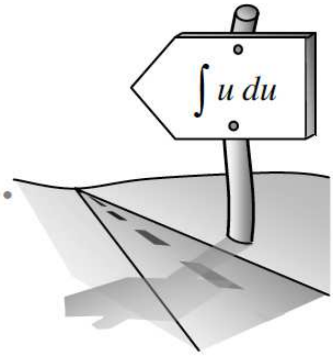
7.2.2 What have u done for me lately?

Integration With U-Substitution



7-67. EASY HARD-LOOKING INTEGRALS, Part Two

Here are some problems similar to those you worked on previously. Again, they have a similar structure. As you work, focus on this question: *How do you decide which constant(s) you will need to multiply by?*

a. $\int 6x^2(x^3+5)\,dx$

b. $\int \frac{3}{4}x^3(x^4+1)^{-1/2}\,dx$

7-68. Antiderivatives (like that of problem 7-67) can get messy because the Chain Rule is involved, only in reverse. At times, it is hard to keep everything straight. You need to keep track of which is the inside function, which is the outside function, and what constants you need to multiply or divide by. What a mess!

One technique that helps organize this work is called substitution. The steps for the substitution method are outlined in the Math Notes box below. Use the steps to solve the following integrals.

a. $\int \sin(x^5)\cdot x^4\,dx$

b. $\int (-3x^4+2)^5\cdot x^3\,dx$

c. $\int \sec(10x^3)\tan(10x^3)\cdot x^2\,dx$

d. $\int x^5(-3+5x^6)^9\,dx$

e. $\int 25x^4\cdot 5^{10x^5+1}\,dx$

f. $\int (x^3+1)(x^4+4x+5)^{8/5}\,dx$

g. Review the problems you just solved using u-substitution. Are there any expressions where more than one possible expression for u could have been defined? Choose at least two to solve again using a different u. Did you get the same answer?

Math Notes

Can you tell just by looking at the integral at right that it is a "reverse Chain Rule" problem? How?

$$\int\left(\tfrac{3}{2}x^4-3x\right)^5\left(2x^3-1\right)dx$$

$$\downarrow$$

$$\int u^5\cdot\left(2x^3-1\right)dx$$

Let's assume the inside function is $\frac{3}{2}x^4-3x$. To clean up the messy integral, let's call the inside function ***u***, and substitute that into the integral.

If $u=\frac{3}{2}x^4-3x$, then

$$\tfrac{du}{dx}=6x^3-3$$

$$du=(6x^3-3)dx$$

Oh-oh! Now we have ***u***' s, x's and dx' s in the same integral. We need to convert everything to ***u***' s.

$$dx=\tfrac{du}{(6x^3-3)}$$

To replace dx, we can differentiate our ***u***equation and solve for dx.

$$\int u^5(2x^3-1)\tfrac{du}{(6x^3-3)}$$

or $\int u^5\cdot\frac{(2x^3-1)}{(6x^3-3)}du$

Now we can substitute this expression in for dx in the integral:

$\frac{(2x^3-1)}{6x^3-3}$ simplifies to $\frac{(2x^3-1)}{3(2x^3-1)}=\frac{1}{3}$

If we chose our ***u*** correctly, the remaining x' s should disappear through simplification.

$$\int\left(\tfrac{3}{2}x^4-3x\right)^5\left(2x^3-1\right)dx$$

$$\int u^5\cdot\tfrac{1}{3}du$$

At last! Now we can write the integral completely in terms of ***u***:

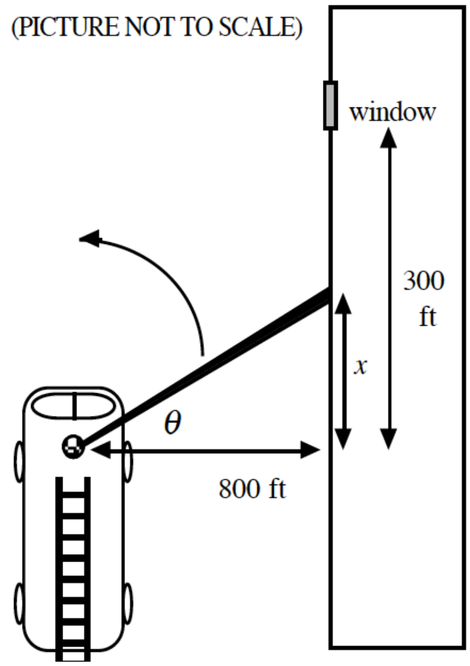
$$\tfrac{1}{18}u^6+C$$

$$\downarrow$$

Now we can integrate:

$$\tfrac{1}{18}\left(\tfrac{3}{2}x^4-3x\right)^6+C$$

And finally, since $u=\frac{3}{2}x^4-3x$, we can substitute this back in for $\boldsymbol{u}$ and simplify:



7-69. As a fire truck is parked outside a building, its rotating red beacon makes a red dot as it shines on the building wall.

a. If the beacon rotates at $\frac{\pi}{10}\,\frac{\text{rad}}{\text{sec}}$, how fast is the red dot moving along the wall when it hits a window 300 feet away from the wall's closest point?

b. Were you surprised by how fast the red dot was moving? Why is that speed so high?

Review & Preview

7-70. Once again, Greta is in a pickle! She is trying to solve the following equation and knows she can use u-substitution to do it: Homework Help ✎

$$3\tan^2 x - \tan x - 10 = 0$$

a. Greta thinks if she lets $u = \tan x$ she can write the problem as a quadratic equation. Show Greta she is correct.

b. Use your answer to part (a) to find the value(s) of x.

7-71. Remember Eric and the 16-foot tall lamppost? If Eric (who is 5 feet tall) walks away from the pole at a rate of $4\,\frac{\text{ft}}{\text{sec}}$, at what rate is the *tip* of his shadow moving away from the lamppost? Draw a diagram before you start this problem. Homework Help ✎

7-72. Find $\frac{dy}{dx}$. Homework Help ✎

a. $y = e^{\sin(2x)}$

b. $y = \ln(\cos(2x))$

c. $y = \ln e^x$

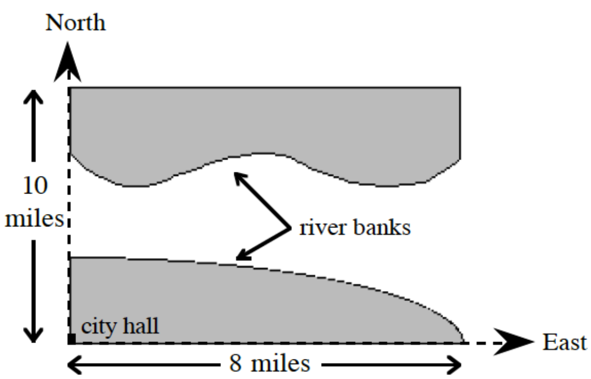
7-73. Integrate. Homework Help ✎

a. $\int 2\sin(2x)e^{\cos(2x)}dx$

b. $-\int (6x-2)^7 dx$

c. $\int \frac{\ln x}{2x} dx$

7-74. The city planning board has set aside funds to build a bridge going north and south across the Newton River. To save money, they have agreed to build the shortest bridge possible. A map for the 8-mile wide city is shown below. The equations below represent the river banks where x is the number of miles east of City Hall and y is the number of miles north of City Hall. Find the location for the shortest bridge that will span the Newton River. Homework Help ✎



North Bank: $y = \cos\left(\frac{\pi x}{2}\right) + 6$
South Bank: $y = \ln(9 - x)$

7-75. No Calculator! Determine where the following functions are discontinuous and/or non-differentiable. Homework Help ✎

a. $f(x) = \begin{cases} \frac{1}{2}x^2 + \frac{3}{2} & \text{for } x \le -1 \\ |3 - x| & \text{for } x > -1 \end{cases}$

b. $g(m)=\begin{cases} \frac{2}{m-3} & \text{for} \quad m<2 \\ -\sqrt{6-m} & \text{for} \quad 2 \le m < 5 \\ \frac{1}{2}m-\frac{7}{2} & \text{for} \quad m \ge 5 \end{cases}$

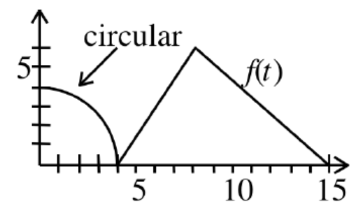
7-76. While chasing a rabbit, a greyhound starts from rest and accelerates at 6 ft/sec^2 until it reaches its maximum speed. Homework Help ✎

a. How long does it take for the greyhound to reach its maximum speed of 30 feet per second?

b. How far did the greyhound have to travel to catch the rabbit if it took a total of 8 seconds to catch it?

7-77. After analyzing a function $f(x)$, Edwin knew that $f'(3) = 0$ and $f''(3) = -2$. He therefore concluded that $f(x)$ has an absolute maximum at $x = 3$. Edwina, his girlfriend, is not so sure. Explain why Edwina is unwilling to accept Edwin's conclusion. Homework Help ✎

7-78. $F(x)=\int_0^x f(t)\,dt$ for the function $f(t)$ graphed at right. Homework Help ✎



a. When is $F(x)$ at a maximum on [0, 15]?

b. Is $F(x)$ increasing or decreasing or both on [0, 8]?

c. Find $F(4)$, $F(10)$, and $F(15)$.

d. List the interval(s) on [0, 15] for which $F''(x) > 0$.