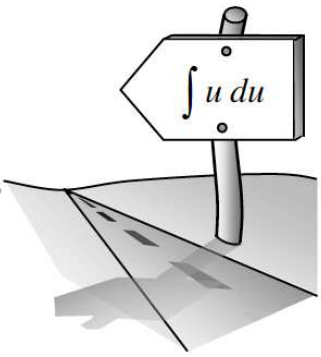
7.2.3 I need *u* I *du*.



Definite Integrals and *u*-Substitution

7-79. Use any of the techniques you have learned to evaluate this antiderivative. Explain and *show* how you know your answer is correct.

$$\int \left(\tfrac{3}{2}x^4 - 3x\right)^5 (2x^3 - 1)dx$$

7-80. Use the substitution method to evaluate the following integrals. State the substitution,*u*, used to evaluate the integral.

a. $\int 6\sin\left(5x^3\right)\cdot x^2 dx$

b. $\int e^{(5x^{10}-2)}\cdot x^9 dx$

c. $\int x^3(7x^4-3)^5 dx$

d. $\int \sin(x^4-3)\cos^5(x^4-3)x^3 dx$

e. $\int \frac{x^2}{x^3-1}dx$

f. $\int \frac{1}{x^2}\sqrt{\frac{1}{x}+1}dx$

7-81. Remember Greta? She is now trying to find the area under $f(x) = x^3\,(6x^4 - 3)^5$ on $-1 < x < 0$. She set up and integral and used *u*-substitution and found that the area is exactly $-\frac{1}{144}$ square units. Her teammate Hansol is convinced that Greta made a mistake.

a. Check Greta's work. If you find an error, explain.

Problem: $\int_{-1}^{0} x^3(6x^4-3)^5\,dx$ and let $u = 6x^4 - 3$

Step 1:

Step 1: $\int_{-1}^{0} \frac{1}{24} u^5 du$

Step 2: $\frac{1}{24}\left(\frac{1}{6}u^6\Big|_{-1}^{0}\right) = -\frac{1}{144}$

b. When Hansol evaluated the integral on her calculator, she found that the actual answer was 0 square units. Greta must have done something wrong! According to Hansol, the integral should be written as so: $\int_{3}^{-3} \frac{1}{24} u^5 \, du$. Explain why Hansol changed the bounds on integration.

c. Evaluate Hansol's integral in terms of u. What is the area under the curve?

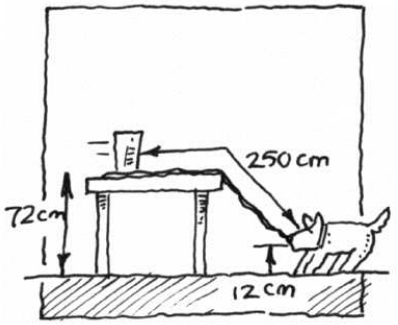
7-82. In the following problems, rewrite the integral in terms of u. Be careful of the bounds. You do not need to solve the problem.

a. $\int_{1}^{3} 2x(x^2+1)^4 \, dx$

b. $\int_{0}^{\pi/4} \sec^2 x(\tan x + 3) dx$

7-83. BAD GIRL!

Ms. D's famous dog, Basil, is very naughty! One day, she grabbed the edge of the tablecloth and backed away at $15 \frac{\text{cm}}{\text{sec}}$. This caused a glass on the table near the other end of the tablecloth to move to the edge and fall off and shatter. Basil's mouth is 12 cm above the floor, the table is 72 cm high, and there is 250 cm of tablecloth between Basil's mouth and the glass. At the instant the glass reached the table's edge, how fast was it traveling?



Review & Preview

7-84. Find $\frac{dy}{dx}$ for the following functions. Homework Help

a. $y = \tan^{-1}(2x)$

b. $y = \tan(2x)$

c. $y = \cot(2x)$

7-85. Integrate. Homework Help

a. $\int -\frac{2}{x}\,dx$

b. $\int \left(\frac{2}{\sqrt{1-x^2}} + 4x \right) dx$

c. $\int_0^{\pi} \pi^x dx$

7-86. Grain pouring from a conveyor belt falls into a pile in the shape of a cone. The grain is falling at a rate of 8 $\frac{\text{m}^3}{\text{min}}$, and forms a pile whose radius is always half the height. How fast is the height of the pile increasing when the radius is 6 meters? Homework Help

7-87. THE WEDDING CAKE, Part Two

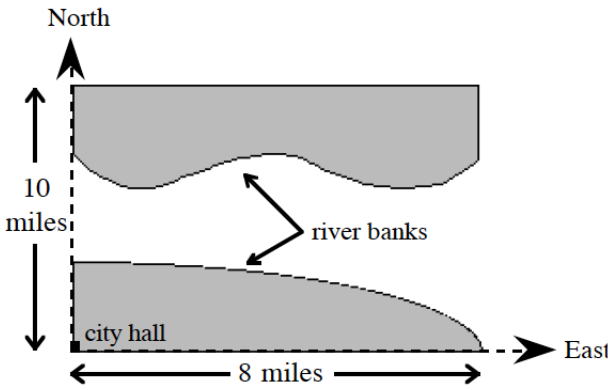
Kiki is still trying to decide on her wedding cake. She decided to keep the same shape and size but instead have 8 layers. Therefore, the diameters of the layers would be 16, 14, 12, 10, 8, 6, 4, and 2 inches. Each layer for this cake would only be 2 inches tall. Homework Help

a. Set up a Riemann sum to evaluate the volume of the cake.

b. Find the volume of this latest design.

7-88. Yong Li's initial deposit of \$1 has grown to \$5 in just 2 years of compounding continuously! What was her annual interest rate? Homework Help

7-89. Remember our city that straddles the Newton River? It is applying for National Parks and Recreation funds and needs to know the area of land (shaded in the diagram) within the city limits.
Given the equations of the river banks, determine the area of land.



North Bank: $y = \cos\left(\frac{\pi x}{2}\right) + 6$

South Bank: $y = \ln(9 - x)$ Homework Help

7-90. The limit below represents a definite integral. Fill in the blank:
Homework Help

$$\lim_{n\to\infty} \frac{6}{n}\left[\left(\frac{1}{n}+3\right)+\left(\frac{2}{n}+3\right)+\ldots+\left(\frac{n}{n}+3\right)\right] = \int_{\square}^{\square} \underline{\qquad} \, dx$$

7-91. Feng throws a baseball directly up in the air with an initial velocity of 16 feet per second from a height of 4 feet. Homework Help ✎

a. When will the ball hit the ground?

b. What is the ball's maximum height?

c. What is the ball's velocity when it hit the ground?

d. What is the total distance the ball traveled from $t = 0$ sec to $t = 0.75$ sec.