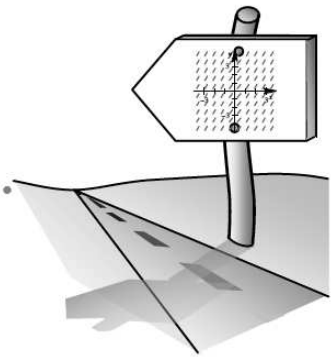
# 7.3.1 Can I solve a diff-EQ?

Solving Differential Equations



**7-104.** If $\frac{dy}{dx} = x$, then what does $y$ equal? Is there more than one possible solution? Show how you got your answer.

**7-105.** Find general solutions for $y$.

a. $\frac{dy}{dx} = 6 \cos x$

b. $\frac{dy}{dx} = x^3$

c. $\frac{dy}{dx} = y$

d. Explain why part (c) does not follow the same pattern as parts (a) and (b) for the general solutions.

## Math Notes


### Solving Differential Equations

The equation $\frac{dy}{dx} = x$ is called a**differential equation** because it involves the derivative (or rate of change). We have seen many of these before and have usually solved them by finding a general antiderivative.

However, there is another way to solve some differential equations: by separating the **differentials** ($dx$ and $dy$) and integrating.

Differentials represent infinitesimal changes of $x$ and $y$. Once they are separated, we can integrate both sides implicitly.

| | | | |
|---|---|---|---|
| **Original equation :** | $\frac{dy}{dx}$ | $=$ | $x$ |
| **Separate the differentials:** | $dy$ | $=$ | $x \cdot dx$ |
| **Integrate both sides:** | $\int dy$ | $=$ | $\int x\,dx$ |
| **Combine the constants:** | $y + C_1$ | $=$ | $\frac{x^2}{2} + C_2$ |
| **Solution :** | $y$ | $=$ | $\frac{x^2}{2} + C$ |

Our solution is a function that will make the original differential equation true. That is, if you differentiate our solution and substitute it for $\frac{dy}{dx}$, you get an identity.

**7-106.** YONG LI RETURNS AGAIN!

After speaking with her financial advisor, Yong Li decided to put her money into a new bank account.

a. Describe what the rate of growth in her balance is dependent on.

b. If her balance is $B$ = \$1000 and the bank offers 10% interest, what is $\frac{dB}{dt}$?

c. Find $\frac{dB}{dt}$ if her balance is $B$ = \$5600 and the bank offers 7.5% interest.

d. Write a differential equation relating $\frac{dB}{dt}$ to balance $B$ and interest $r$ = 15%.

**7-107.** SOLVING DIFFERENTIAL EQUATIONS WITH IMPLICIT INTEGRATION

Back in Chapter 6, we used exponential equations to represent Yong Li's bank balance at time $t$. However, our equation from part (d) of problem 7-106 does not look exponential! What is going on? Let's examine how our differential equation (such as $\frac{dB}{dt} = 0.15B$) relates to the exponential function we are used to. Apply the implicit integration technique from problem 7-105 to our differential equation from part (d) of problem 7-106. As you follow the given solution of this differential equation, answer the questions below.

a. In order to solve $\frac{dB}{dt} = 0.15B$, we separate the differentials. Why can we do this?

b. The expression $\int 0.15B \cdot dt$ cannot be evaluated directly because we do not know $B$' s relationship with $t$. In other words, $B$ is defined *implicitly* in terms of $t$. Therefore, alter the equation so that the $B$ and $dB$ are together and $t$ and $dt$ are together. Then integrate both sides.

c. To solve for $B$, we need to "undo" the natural logarithm. How can we do this?

d. Why is an absolute value unnecessary?

e. Explain why $e^{0.15t+c}$ can be replaced with $e^{0.15t}e^{C}$. Also, why can $e^{C}$ be replaced by $C$?

f. Examine the resulting equation for $B$, the balance of Yong Li's bank account after $t$ years. Where is the interest rate represented in this equation? What does the constant represent?

g. Explain why the general solution to Yong Li's problem is $B = Ce^{0.15t}$. Why does $B = e^{0.15t} + C$ not work? Use the derivative to justify your answer.

h. Write a differential equation for a bank account that grows with an interest rate of 35%. Integrate implicitly. Then determine how many years it will take the original balance to double.

$$\frac{dB}{dt} = 0.15B$$

$$dB = 0.15B \cdot dt$$

$$\frac{dB}{B} = 0.15dt$$

$$\int \frac{dB}{B} = \int 0.15dt$$

$$\ln|B| = 0.15t + C$$

$$|B| = e^{0.15t+C}$$

$$B = e^{0.15t}e^{C}$$

$$B = Ce^{0.15t}$$

## Review & Preview

**7-108.** Find the antiderivative of the following differential equations. Use implicit integration when necessary. Solve your equations for $y$. Check your answer by differentiating the result. Homework Help ✎

a. $\frac{dy}{dx} = 7x$

b. $\frac{dy}{dx} = 7y$

c. $\frac{dy}{dx} = e^{-y}$

**7-109.** Nick and Roza are arguing over an implicit integration question. Nick thinks the constant of integration must be added in when you find the antiderivative. Roza thinks you can add the $C$ later, after you have solved for $y$. Which student is correct and why? Give an example to justify your answer. Homework Help ✎

**7-110.** No Calculator! Integrate. Homework Help ✎

a. $\int (3e^{4x} + 2^{x})dx$

b. $\int_0^{\pi} (\sin^3 x \cos x)dx$

c. $\int x^2 (3x^3 + 5)^4 \, dx$

d. $\int \frac{5x^{0.25}}{4x^{1.25}-6}dx$

e. $\int_e^{e^3} \frac{1}{x} \ln x \, dx$

f. $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$

**7-111.** Population growth is proportional to the current population. For example, if a town has 100 people, its change in population for a year will be based on those 100 people. Therefore, its change will be much smaller than that of a large city like San Francisco, with population 800,000. Why? Write down your reasoning. Homework Help ✎

**7-112.** THE WEDDING CAKE, Part Three

Kiki is still trying to decide the shape of her wedding cake. She is now considering having 16 layers! The largest diameter would be 16 inches and each subsequent diameter would decrease by 1 inch. Each layer of this cake would only be 1 inch tall. Homework Help ✎

a. Setup a Riemann sum to calculate the volume of the cake.

b. Find the volume of this cake.

**7-113.** Simplify these exponential expressions. Try each without a calculator first. Homework Help ✎

a. $e^{\ln(11)}$

b. $\ln(e^4)$

c. $\ln(e^e)$

d. $10^{\log(x)}$

**7-114. Multiple Choice:** The average value of $f(x) = e^{\sin x^2}$ on the interval $-1 \le x \le 1$ is nearest to: Homework Help ✎

a. $-1.253$

b. $-1.414$

c. 0

d. 1.253

e. 1.414

**7-115. Multiple Choice:** Find the area of the largest rectangle that can be inscribed under the graph of $y = \cos x$, in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. 7-115 HW eTool (Desmos). Homework Help ✎

a. 1.10

b. 1.12

c. 1.14

d. 1.16

e. 1.18

**7-116. Multiple Choice:** The total area enclosed between the graphs of $y = 3 \sin x$ and $y = x + 1$ is closest to: 7-116 HW eTool (Desmos). Homework Help ✎

a. 5.2

b. 5.3

c. 5.4

d. 5.5

e. 5.6