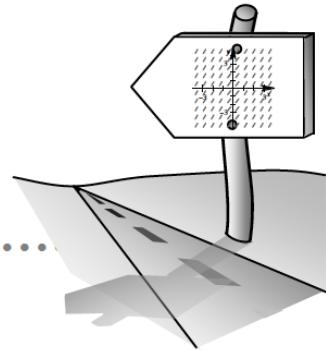


7.3.2 What does warm soda have to do with differential equations?

The Soda Lab: Newton's Law of Cooling



7-117. THE WARM SODA, Part One

After school one day, Liliana, Katrina, and Soraya worked on homework together.



"Yuck! This drink is nasty!" Liliana complained.

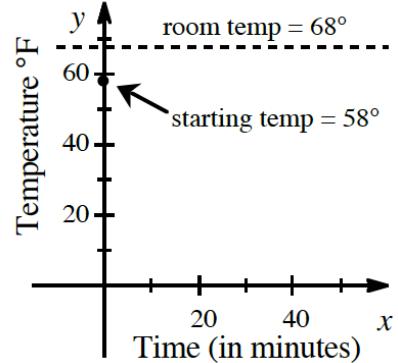
"What's wrong?" asked Soraya.

"It's too warm! It's been sitting out for an hour." replied Liliana. "I would have drank it sooner but I was too wrapped up in this fun calculus problem. I didn't think my soda would warm up that fast!"

"How fast *does* a soda warm up?" Soraya inquired.

"I don't know... let's find out with your soda!" said Liliana as she whipped out a thermometer. She quickly determined that Soraya's soda was currently 58°F .

- Predict how long you think it would take Soraya's soda to warm up to room temperature (68°F).
- Sketch a graph using the axes below to show how you believe the temperature will change over time.
- What type of function would best model this situation?
- An hour later, the temperature of the soda was 63° . Katrina, who was watching this whole experiment from a distance, predicted that the soda would be room temperature after another hour. "You see, it took one hour for the temperature to raise 5° . Therefore, it will take another hour to rise the remaining 5° to room temperature," she explains. Do you agree or disagree with Katrina? Why?
- If the rate of the warming is not constant, then there must be some reason why the rate would change over time. What is the rate of the temperature change dependent upon? How does the temperature of the room affect the rate of temperature change?



- f. The rate of $^{\circ}\text{F}$ the temperature is changing with respect to time is called $\frac{dF}{dt}$. This value is proportional to the difference in temperature between the room and the soda at any given time. Write a rate statement that describes how the rate changes over time. Use k as your constant of proportionality.

7-118. SODA LAB

Study the way the temperature of soda changes over time. Carefully measure the temperature of a cold can of soda for 30 minutes.



- Plot the data on your graphing calculator. Decide what curve would best fit this data and then find a function of best fit.
- Examine the graph of your data. What happens to the soda's temperature as time increases? Why?
- Describe what happens to the rate at which the temperature changes. Does the temperature change at a constant rate? Write a "rate statement" for this relationship.
- Using a table or a spreadsheet, compare the change in temperature with the difference between the soda temperature and the room temperature. Are they related? How?

7-119. THE WARM SODA, Part Two

In problem 7-117, Soraya asked an important question: "How fast does the soda warm up?" Soraya is asking about a *rate*. Therefore, instead of writing a temperature equation ($F = \dots$), we will write a rate equation ($\frac{dF}{dt} = \dots$). Look again at our data from problem 7-118.

- Using the data, set up a differential equation for the rate the temperature of the soda is changing. That is, write an equation that models $\frac{dF}{dt}$, the change in temperature over time.
- Integrate the differential equation to find an equation for temperature at time t .



7-120. Find the antiderivative of the following differential equations. Use implicit integration when necessary. Solve your equations for y . Be careful about introducing the constant of integration at the appropriate time. [Homework Help](#)

a. $\frac{dy}{dx} = 7x + 3$

b. $\frac{dy}{dx} = 7y + 3$

c. $\frac{dy}{dx} = 7y^2$

d. $\frac{dy}{dx} = 7$

e. $\frac{dy}{dx} = e^y$

7-121. Which of the following differential equations could model a population whose rate of change is proportional to the existing population? Explain your reasoning. Is it possible that there is more than one answer? [Homework Help](#) 

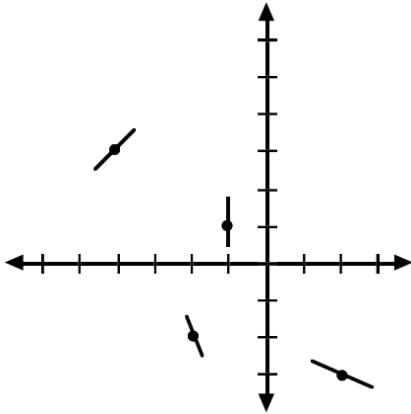
a. $\frac{dP}{dt} = 3P$

b. $\frac{dP}{dt} = 3t$

c. $\frac{dP}{dt} = -3P$

d. $\frac{dP}{dt} = 3$

7-122. Remember Theresa, the girl who loves tangents. Well, Theresa is at it again! She has drawn tangents to her function $g(x)$ and then erased the function. [Homework Help](#) 



a. Trace her diagram on your paper. Using the tangents, sketch a possible function $g(x)$.

b. Is your function differentiable everywhere? Justify your answer.

c. Could any of the points shown be the location of a local maximum or minimum? Justify your ideas.

7-123. Integrate. [Homework Help](#) 

a. $\int \cos x (\sin x)^2 dx$

b. $\int -\cos(3x - 2) dx$

c. $\int x^{-4} e^{x^{-3}} dx$

d. $\int (e^{7x-2} - \sqrt{5x}) dx$

e. $\int 2 \ln(4x) dx$

f. $\int \frac{9}{9+x} dx$

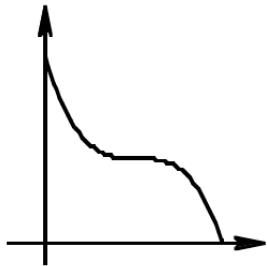
7-124. Differentiate the following equations. [Homework Help](#) 

a. $f(x) = \frac{\tan x}{e^{-x}}$

b. $y^3 - 3xy = 3^y$

c. $g(t) = \ln(t) \sqrt{t}$

7-125. If the graph below represents distance in miles over time between a space shuttle and Earth, how would you find the average velocity? Describe what method you would use. [Homework Help](#) 



7-126. What if the graph above represents the velocity in miles per hour of the shuttle? In this case, how would you find the average velocity? Describe what method you would use. [Homework Help](#) 