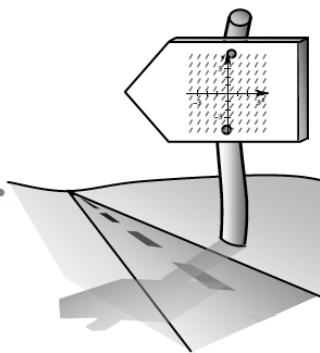


7.3.3 How do I draw a slope field?

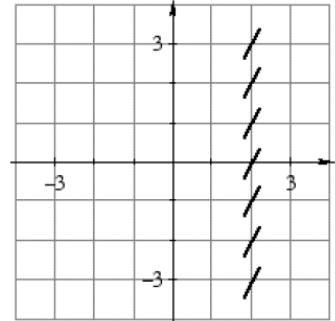
Slope Fields with Parallel Tangents



7-127. GRAPHS OF DIFFERENTIAL EQUATIONS

Theresa (who does not know how to integrate) is studying the differential equation $\frac{dy}{dx} = x$. She knows that this differential equation represents the slopes of the tangent lines of another function, y . She wonders what y looks like on the graph.

- Theresa started to sketch. She drew tiny tangent lines where $x = 2$. She was very careful to draw them with the correct slope. Observe her sketch. Why do all the tangent lines have the same slope at $x = 2$?
- Complete the graph on the [Lesson 7.3.3A and 7.3.3B Resource Pages](#) provided by your teacher. Draw small tangent lines through every point on the domain $[-3, 3]$. This type of graph is referred to as a slope field.
- Theresa starts jumping with joy. "My slope field," she exclaims, "reveals that the anti-derivative of $\frac{dy}{dx} = x$ is $y = \frac{1}{2}x^2 + C$!" Use differentiation to explain how she knew.
- Theresa is ecstatic because there are so many "solutions" to the differential equation. What does she mean by solution? How many are there?
- Theresa wondered about the solution that goes through $(-3, 1)$. How many solutions go through her favorite point? Sketch these "particular solutions."



7-128. Theresa is not done. She wonders about other slope fields for $\frac{dy}{dx} = y$. On the same [Lesson 7.3.3A and 7.3.3B Resource Pages](#) you used for 7-127, sketch slope fields for the following differential equations.

a. $\frac{dy}{dx} = -2y$

Compare and contrast it to the slope field in problem 7-127. Explain this relationship.

b. $\frac{dy}{dx} = x + y$

Compare and contrast this slope field to the other two you have done.

c. $\frac{dy}{dx} = \frac{y}{x}$

Be sure to calculate the slope at each point with the appropriate x - and y -values at each point.

7-129. USING SLOPE FIELDS TO GRAPH SOLUTIONS

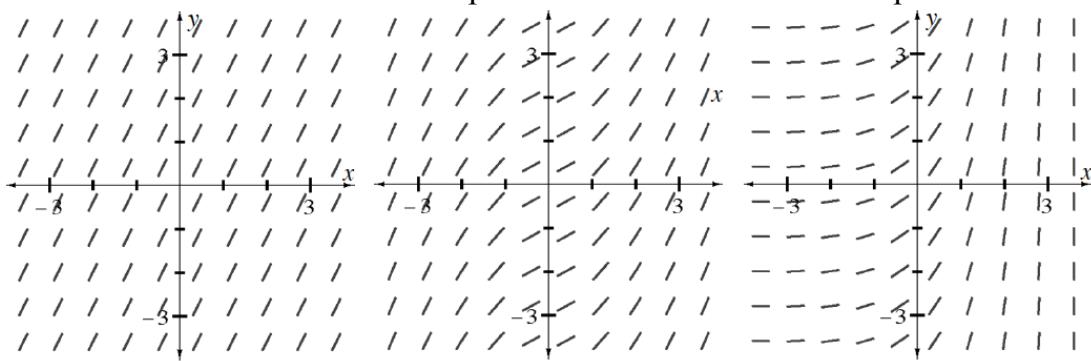
a. Solve the following differential equations.

i. $f'(x) = e^x$

ii. $\frac{dy}{dx} = 2$

iii. $\frac{dy}{dx} = x^{2/3}$

b. The slope fields for the differential equations in part (a) are shown below. On your [Lesson 7.3.3A](#) and [7.3.3B Resource Pages](#), label each slope field with its differential equation and general solution. Then sketch at least two particular solutions on each slope field.



c. For each of the slope fields above, the tangents appear to be parallel at each x -value. Why do you think that is? Look at the differential equations to help guide your answer.

7-130. Steady Stephanie draws her slope fields so slowly that it is hard for her to finish her assignment. A graphing calculator program or computer simulator could speed up the process. Your team will use technology to explore the slope fields of the parent graphs below. On your [Lesson 7.3.3A](#) and [7.3.3B Resource Pages](#):

- Roughly sketch each of the slope fields.
- Draw at least two solutions.
- Solve the differential equation and compare the results.



Each team member should be prepared to share the results of the graphs they explored to the other members of the team.

a. $\frac{dy}{dx} = x$

b. $\frac{dy}{dx} = x^2$

c. $\frac{dy}{dx} = x^3$

d. $\frac{dy}{dx} = e^x$

e. $\frac{dy}{dx} = \ln x$

f. $\frac{dy}{dx} = \sqrt{x}$

g. $\frac{dy}{dx} = \sin x$

h. $\frac{dy}{dx} = \tan x$

i. $\frac{dy}{dx} = |x|$

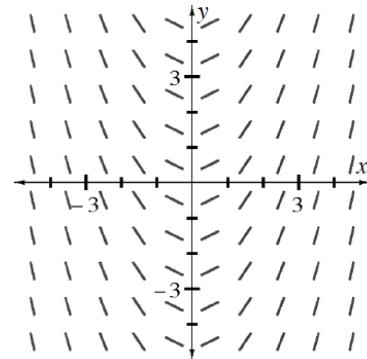
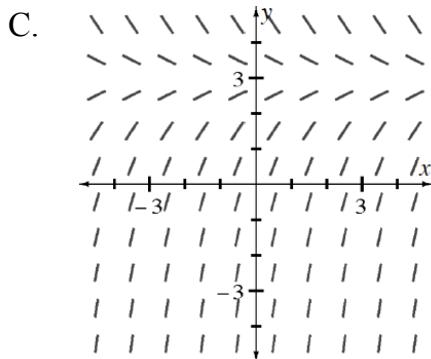
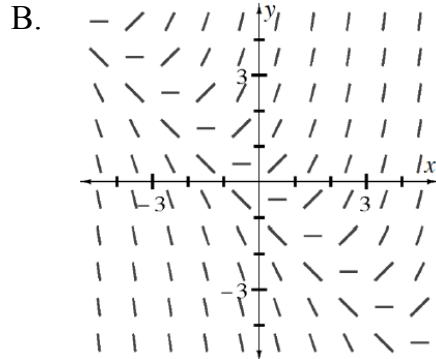
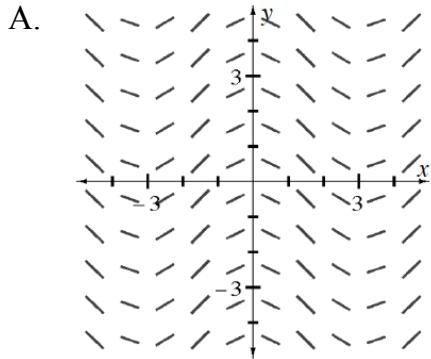
7-131. Roberto drew the slope fields below but forgot to label them with their differential equations. His teammates, Min and Arak, decided to help him out, but they are not sure how to proceed. They do know that the differential equations are:

$$\frac{dy}{dx} = 3 - y$$

$$\frac{dy}{dx} = x + y$$

$$\frac{dy}{dx} = x$$

$$\frac{dy}{dx} = -\sin x$$

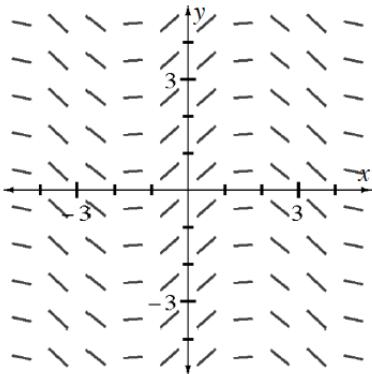


- Arak suggests using implicit integration to solve each differential equation and find a match. Explain Arak's method.
- Min says, "All you have to do is pick a few coordinate points and plug them in to the differential equation." Explain Min's method.
- Roberto thinks that Arak's method and Min's method would take too much time. "All you have to do is look at the orientation of the parallel tangent lines!" says Roberto. Explain Roberto's method.
- Use any method you choose to label each slope field with its differential equation.
- Explain why graph (d) does not have parallel tangent lines.

7-132s. Determine the particular solution to the differential equation $\frac{dy}{dx} = \frac{-3y}{x}$ given the initial condition $f(1) = -1$ and state its domain.



7-132. Study the slope field for $\frac{dy}{dx}$ below. Use it to visualize a possible function for y . [Homework Help](#)



- What type of functions are y and $\frac{dy}{dx}$?
- Sketch the particular solution of y through the point $(0, 0)$ on your own set of axes. Justify your answer.

7-133. Set up each integral in terms of u . Be careful of the bounds. You do not need to solve the problem. [Homework Help](#)

a. $\int_0^2 xe^{3x^2} dx$

b. $\int_{-2}^3 \frac{5x^2}{x^3+1} dx$

7-134. Find the equation of a line tangent to the graph of $f(x) = \arcsin x$ at $x = \frac{1}{2}$. [Homework Help](#)

- Use the tangent line to find an approximate value of $f(0.52)$.
- Is your approximation an over or under estimate? Use the second derivative to justify your answer.

7-135. Evaluate each of the following integrals. [Homework Help](#)

a. $\int_0^{1/6} \frac{1}{\sqrt{1-9x^2}} dx$

b. $\int_0^{\sqrt{3}/2} \frac{1}{1+4x^2} dx$

c. $\int_0^1 2^{3x} dx$



d. $\int_0^1 e^{5x} dx$

7-136. Find the area between the curves $f(x) = (x - 1)^3$ and $g(x) = x - 1$. [Homework Help](#)

7-137. Given: $f(x)$ and $g(x)$ continuous and differentiable such that $f(g(x)) = x$. [Homework Help](#)

Evaluate.

a. $f'(g(0))$

b. $g'(1)$

c. $g'(2)$

d. $f'(g(2))$

e. $3 \cdot f'(2)$

f. $5 \cdot f'(1) + 6 \cdot g'(1)$

x	$f(x)$	$f'(x)$	$g(x)$
-1	2	1	0
0	-1	2	2
1	1	7	1
2	0	3	-1

7-138. Find all relative maxima, minima, and points of inflection given $f(x) = 2x^{5/3} - 5x^{4/3}$. [Homework Help](#)

7-139. Evaluate each limit. [Homework Help](#)

a. $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+2}}$

b. $\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2+2}}$

c. $\lim_{x \rightarrow 2} \frac{\frac{1}{x}-\frac{1}{2}}{x^2-4}$