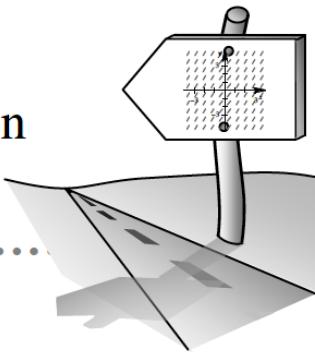


## 7.3.4 When are solutions parallel and when are they not?

### Slope Fields with Non-Parallel Tangents



**7-140.** Given  $\frac{dy}{dx} = -\frac{x}{y}$ .

- Sketch the slope field.
- Do the slopes seem to behave in any special way? Why?
- Sketch the particular solution to the differential equation that passes through  $(0, 1)$ .
- Find the solution,  $y$ , to the differential equation that passes through  $(0, 1)$ . Use the point to solve for  $C$ , the constant of integration.

**7-141.** Discuss with your team how to quickly plot the following slope fields. Will the tangent lines be parallel? If so, in which direction? Then sketch each on your own paper.

a.  $\frac{dy}{dx} = y + 1$

b.  $\frac{dy}{dx} = x + 1$

- c. Find the general solution to each differential equation using implicit integration. Discuss any similarities and differences.

### MATH NOTES



### Differential Equations and Slope Fields

So far in this course, we have interpreted integrals in two ways:

- As an antiderivative:  $F(x) = \int f(x) dx$
- As an area function:  $A(x) = \int_c^x f(x) dx$

However, there is another important interpretation of an integral. It is the view of an integral as a **solution to a differential equation**. A differential equation is one which defines a derivative, such as:

$$f'(x) = 1 \text{ or } \frac{dy}{dx} = -xy + y^2$$

The solution is the function  $f(x)$  whose slope is  $f'(x)$  for all  $x$ . This is written:

$$f(x) = \int f'(x) dx$$

Because integration produces a constant, there are an infinite number of solutions to a differential equation unless additional information is given. A useful tool to determine the shape of a solution is a **slope field**, or direction field. This is a graph consisting of possible tangents to a function  $y$  given the differential equation  $\frac{dy}{dx}$ . The tangents help our eye determine the shape of  $y$ .

For example, the equation  $\frac{dy}{dx} = 1$  indicates that for all  $y$ , the slope of  $y$  is 1. Therefore, the slope field would show tangents with  $m = 1$  for all points.

### 7-142. THE SLOPE FIELD SORT

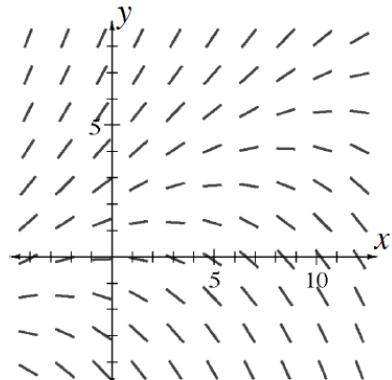
Your teacher will give you a slope field [Lesson 7.3.4A, B, and C Resource Pages](#) including two sets of cards. Your task is to match each graph with its differential equation and its general solution. Be sure to discuss your choices with your team and explain your reasoning when other team members disagree. Do not use a calculator!



### 7-143. DO ALL SOLUTIONS LOOK THE SAME?

The slope field at right represents the differential equation

$$\frac{dy}{dx} = -0.1x + 0.2y.$$



- On your paper, plot a particular solution containing the point  $(0, 1)$ . Plot another solution containing  $(0, 4)$ .
- For this slope field, the different solutions do *not* look the same. What happened?



**7-144.** For each of the functions  $f'(x)$  defined below, find *two* different possible functions for  $f(x)$ . [Homework Help](#)

a.  $f'(x) = 1$

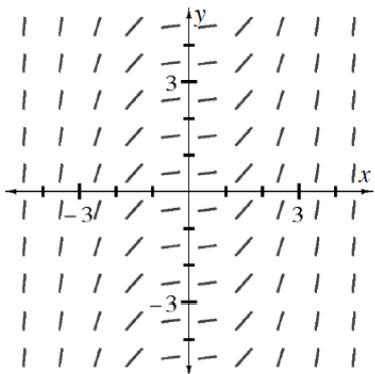
b.  $f'(x) = x^2$

**7-145.** Use the identity  $\cos(2x) = 2 \cos^2 x - 1$  to evaluate  $\int 4 \cos^2 x \, dx$ . [Homework Help](#)

**7-146.** In order to find the average value of a function, sometimes it makes sense to integrate while other times we find the slope of a secant line. When do we use each strategy? [Homework Help](#)

**7-147.** Find a possible solution for  $y$  if  $\frac{dy}{dx} = 0.5x^2$ .

Then use the slope field for  $\frac{dy}{dx}$  below to help graph a family of functions for  $y$  (place your paper over the slope field and use the tangents as guides). [Homework Help](#)



**7-148.** Let  $y(t)$  denote the temperature (in °F) of a cup of tea at time  $t$  (in minutes). The temperature of the tea starts at 190°, while the room temperature is 70°. The tea's change in temperature is described by the equation: [Homework Help](#)

$$\frac{dy}{dt} = -0.1(y - 70)$$

- a. Describe the change in temperature of the tea in relation to the room temperature.
- b. How hot is the tea at any time  $t$ ?
- c. How hot is the tea after 10 minutes?

**7-149. Multiple Choice:** Two particles start at the origin and move along the  $x$ -axis. For  $0 \leq t \leq 2\pi$ , the position functions are given by  $x_1 = -\cos(2t)$  and  $x_2 = e^{(t-1)/4} - 0.5$ . For how many values of  $t$  do the particles have the same velocity? [Homework Help](#)

a. 0

b. 1

c. 2

d. 3

e. 4

**7-150. Multiple Choice:** The area of the region bounded above by the curve  $y = \arctan x$  and below by the curve  $y = x^2 + 3x$  is approximately: [Homework Help](#) ↗

a. 2.06

b. 2.12

c. 2.18

d. 2.24

e. 2.30

**7-151. Multiple Choice:** If  $y = x + \sin(xy)$ , then  $\frac{dy}{dx} =$

[Homework Help](#) ↗

a.  $1 + \cos(xy)$

b.  $1 + y \cos(xy)$

c.  $\frac{1}{1-\cos(xy)}$

d.  $\frac{1}{1-x \cos(xy)}$

e.  $\frac{1+y \cos(xy)}{1-x \cos(xy)}$