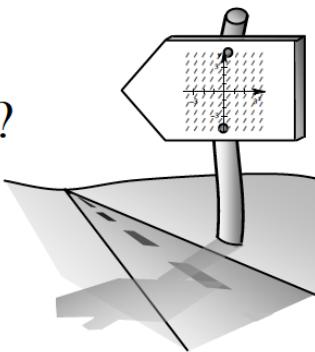


## 7.3.5 When do I use differential equations?

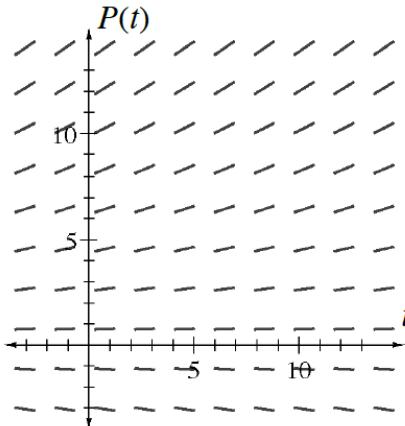
### Differential Equation and Slope Field Applications



**7-152.** Since  $\frac{d}{dx} [\ln x] = \frac{1}{x}$ , then it seems that  $\int \frac{1}{x} dx$  should equal  $\ln(x) + C$ . However, in Chapter 6, we were told that  $\int \frac{1}{x} dx = \ln|x| + C$ . Why is this? Use a slope field for  $\frac{dy}{dx} = \frac{1}{x}$  to investigate why the absolute value is necessary. Write down your observations.

#### 7-153. POPULATION EXPLOSION!

In 2008 San Francisco's population was approximately 800,000 people. If its population grows at a rate of 5% per year, then San Francisco could expect to increase by 40,000 people in 2009. In 2010, the increase should be 5% of its *new* population (840,000), which is 42,000. Therefore, each year as the population changes, the rate of change,  $\frac{dP}{dt}$ , changes.



- Write an equation that represents  $\frac{dP}{dt}$ , the rate of change of the population with respect to time.
- Study the slope field at right for San Francisco. The slope of each tangent line represents the rate of growth for  $P(t)$ . Examine the tangent lines for  $P = 800,000$ . Why do they all have the same slope?
- Place your paper over the slope field. If  $P(0) = 800,000$ , draw the particular solution of  $P$  given this *initial condition*. What type of function is  $P(t)$ ?
- Use implicit integration to find  $P(t)$ .
- The slopes of  $P$  are not the same for each value of  $t$ , yet depend only on the values of  $t$ . Explain why. Hint: Think about the role of the constant of integration in this problem versus other problems.
- Set up an equation that would estimate future populations of San Francisco if the city grows at a rate of 3.5% per year. Use this equation, and the fact that the 2008 population is 800,000 to estimate the population in the year 2099.

**7-154.** Set up a differential equation to express the information below. Define the variables you use.

- The rate at which the population of Los Angeles changes varies directly with its current population.

- b. The rate of change of the volume of water in a tank is proportional to the difference between the amount entering and the amount leaving.

### 7-155. POMP AND CIRCUMSTANCE

Today is graduation day and Winnie awoke from a foggy dream at 8:30 a.m. Struggling to be alert, she made herself a cup of coffee. She remembered that she had to be out of the house at 9:30 a.m. to make it to school on time. When she tasted the cup of coffee, she burned her mouth. "No wonder," she thought after testing its temperature, "this coffee is 205°F!" She put the coffee on the counter and ran off to get dressed. Forty-five minutes later, she returned to find her coffee lukewarm (in fact, it was 98°F). "At least it's not room temperature," she thought, since she knew the room temperature was 68°F. She left to continue preparing her hair.



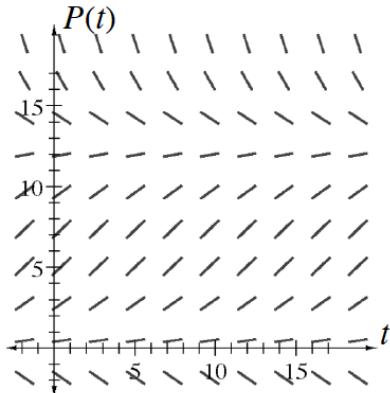
Later, with her hair up and her shoes on, she was finally ready! However, she could not see the clock without her glasses! One last swig of coffee revealed it was now cold: 82°F. Winnie was worried that she was going to be late to her graduation. Was she late? Use a differential equation to determine your answer.



- 7-156.** Stingray populations grow based on the differential equation below, where  $P$  is the population (in thousands) and  $t$  is time in years. Use the graph of the slope field at right to answer the following questions. [Homework Help](#)

$$\frac{dP}{dt} = 0.026P(12.5 - P)$$

- If there are 3,000 stingrays at time  $t = 0$ , sketch a curve representing their population of stingrays.
- What if the original population at  $t = 0$  is 10,000 stingrays? Draw that population curve and decide if its growth is the same or different as that in part (a).
- What if the original population at  $t = 0$  is 17,000 stingrays? Draw that population curve and decide if its growth is the same or different as those in parts (a) and (b).



- 7-157.** Integrate. Show your steps—if you use  $u$ -substitution be sure to change the bounds of integration. [Homework Help](#)

$$a. \int_{\pi/4}^0 \sqrt{\sin x} \cdot \cos x \, dx$$

$$b. \int \frac{1}{\sin x} \cos x \, dx$$

c.  $\int \frac{3x^4 - 4x^2 - 11x + 6}{x^2} dx$

d.  $\int \frac{1}{|x|\sqrt{x^2 - 1}} dx$

e.  $\int_1^e \frac{1}{x} \cos(\ln x) dx$

f.  $\int_{-1}^3 x^2(x^3 - 8) dx$

**7-158.** Explain why a differential equation has infinitely many solutions. [Homework Help](#) 

**7-159.** A Ferris wheel, 50 feet in diameter, revolves at a rate of 2 radians per minute. How fast is a passenger moving when the passenger is 15 feet higher than the center of the Ferris wheel and is rising?

[Homework Help](#) 

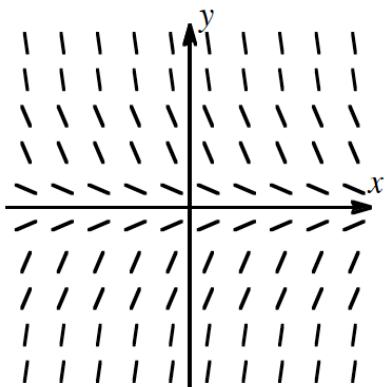
**7-160.** During the semester, Arezo's daily grade at time  $t$  (in days) is  $G(t) = 0.008t^2 + 22 \sin\left(\frac{t}{5}\right) + 45$ . She will receive \$50 if she passes calculus, which requires an average grade of 60% or better. If her semester grade (over 18 weeks) is the average grade during that time period, should she celebrate?

[Homework Help](#) 



**7-161. CHECK FOR UNDERSTANDING: SLOPE FIELDS**

When you draw a function based on a slope field, what are you finding? What is its relationship with the equation that formed the slope field to begin with? [Homework Help](#) 



**7-162.** No calculators! Suppose  $\frac{dy}{dx} = \frac{3x^2}{e^y}$ . [Homework Help](#) 



- Find the particular solution of this differential equation containing the point  $(0,3)$ .
- If you have not already done so, solve your equation for  $y$ .
- Find the domain and range of your equation.

**7-163. Multiple Choice:** The normal line to the curve represented by the equation  $y = x^2 + 6x + 4$  at the point  $(-2, -4)$  also intersects the curve at  $x = ?$  [Homework Help](#)

a.  $-6$

b.  $-\frac{9}{2}$

c.  $-\frac{7}{2}$

d.  $-3$

e.  $-\frac{1}{2}$

**7-164. Multiple Choice:** A point  $(x, y)$  is moving along a curve  $y = f(x)$ . At the instant when the slope of the curve is  $-\frac{1}{3}$ , the  $x$ -coordinate of the point is increasing at a rate of 5 units per second. The rate of change, in units per second, of the  $y$ -coordinate of the point is: [Homework Help](#)

a.  $-\frac{5}{3}$

b.  $-\frac{1}{3}$

c.  $\frac{1}{3}$

d.  $\frac{5}{3}$

e.  $\frac{3}{5}$