

## 8.1.2 How can I predict the graph?

### More Graphs of Polynomials



Today you will use what you learned in the Polynomial Function Investigation in Lesson 8.1.1 to respond to some questions. Thinking about how to answer these questions should help you clarify and expand on some of your ideas as well as help you learn how to use the polynomial vocabulary.

**8-26.** As directed by your teacher, use your finger to trace an approximate graph of polynomial functions in the air. Alternatively, you may sketch each of the polynomial functions below quickly on paper. Just sketch the graph without the  $x$ - and  $y$ -axes.

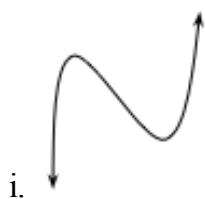
- $P(x) = (x + 10)(x + 7)(x - 12)$
- $Q(x) = (x + 6)(x + 3)(x - 5)(x - 8)$
- $R(x) = -(x + 4)(x + 2)(x - 6)(x - 10)$
- $W(x) = (x + 7)^2(x - 7)^2$
- $S(x) = (x + 6)(x + 3)(x - 5)(x - 8)(x - 12)$



**8-27.** Look back at the work you did in Lesson 8.1.1 problem 8-2, Polynomial Function Investigation. Then answer the following questions.

- What is the maximum number of roots a polynomial of degree 3 can have? Sketch an example.
- What do you think is the maximum number of roots a polynomial of degree  $n$  can have?
- Can a polynomial of degree  $n$  have fewer than  $n$  roots? Under what conditions?

**8-28.** For each polynomial function shown below, state the minimum degree its equation could have.





- Which of the graphs above show that as the  $x$ -values get very large, the  $y$ -values continue to get larger and larger?
- How would you describe the other graphs for very large  $x$ -values?
- When the  $y$ -values of a graph get very large as the  $x$ -values get large, the graph has **positive orientation**. When the  $y$ -values of a graph get very small as the  $x$ -values get large, the graph has **negative orientation**. How is each of the above graphs oriented?

**8-29.** For each graph in problem 8-28, you decided what the minimum degree of its equation could be. Under what circumstances could graphs that look the same as these have polynomial equations of a *higher* degree?

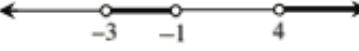
Consider the graphs of  $y = (x - 1)^2$  and  $y = (x - 1)^4$

- How are these graphs similar? How are they different?
- Could the equation for graph (ii) from the previous problem have degree 4?
- Could it have degree 5? Explain.
- How is the graph of  $y = x^3$  similar to or different from the graph of  $y = x^5$ ?
- How do the shapes of graphs of  $y = (x - 2)^3$  and  $y = (x + 1)^5$  with repeated factors differ from the shapes of graphs of equations that have three or five factors that are different from one another?

**8-30.** In the first example from the Polynomial Function Investigation,  $P_1(x) = (x - 2)(x + 5)^2$ ,  $(x + 5)^2$  is a factor. This squared factor produces what is called a **double root** of the function.

- What effect does this have on the graph?
- Check your equations for a **triple root**. What effect does a triple root have on the graph?

**8-31.** You can use a number line to represent the  $x$ -values for which a polynomial graph is above or below the  $x$ -axis. The bold parts of each number line below show where the output values of a polynomial function are positive. That is, where the graph is above the  $x$ -axis. The open circles show locations of the  $x$ -intercepts or roots of the function. Where there is no shading, the value of the function is negative. Sketch a possible graph to fit each number line, and then write a possible equation. Each number line represents the  $x$ -axis for a different polynomial.

- a. 
- b. 
- c. 
- d. 

**8-32.** What can you say about the graphs of polynomial functions with an even degree compared to the graphs of polynomial functions with an odd degree? Use graphs from the Polynomial Functions Investigation (and maybe some others), to justify your response.

**8-33.** Choose three of the polynomials you graphed in the Polynomial Functions Investigation (problem 8-2) and create number lines for their graphs similar to the ones in problem 8-31.

$$P_1(x) = (x - 2)(x + 5)^2$$

$$P_2(x) = 2(x - 2)(x + 2)(x - 3)$$

$$P_3(x) = x^4 - 21x^2 + 20x$$

$$P_4(x) = (x + 3)^2(x + 1)(x - 1)(x - 5)$$

$$P_5(x) = -0.1x(x + 4)^3$$

$$P_6(x) = x^4 - 9x^2$$

$$P_7(x) = 0.2x(x + 1)(x - 3)(x + 4)$$

$$P_8(x) = x^4 - 4x^3 - 3x^2 + 10x + 8$$

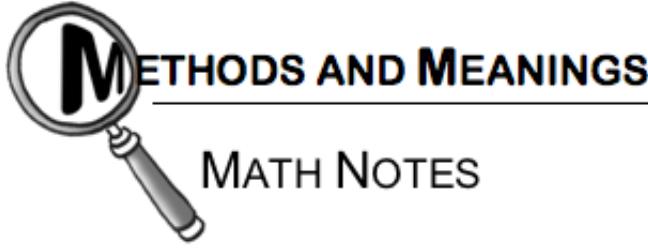
**8-34.** Create a new number-line description (like the ones in problem 8-31) and then trade with a partner. (Each team member should create a different number line.) After you have traded, find a possible graph and equation for a polynomial function to fit the description you have received. Then justify your results to your team and check your team members' results.

**8-35.** Without using a calculator, sketch rough graphs of the following functions.

a.  $P(x) = -x(x + 1)(x - 3)$

b.  $P(x) = (x - 1)^2(x + 2)(x - 4)$

c.  $P(x) = (x + 2)^3(x - 4)$



## Roots and Zeros

The **roots** of a polynomial function,  $p(x)$ , are the **solutions** of the equation  $p(x) = 0$ . Another name for the roots of a function is **zeros of the function** because at each root, the value of the function is zero. The real roots (or zeros) of a function have the same value as the  $x$ -values of the  $x$ -intercepts of its graph because the  $x$ -intercepts are the points where the  $y$ -value of the function is zero.

Sometimes roots can be found by factoring and solving for  $p(x) = 0$ .

In the Parabola Lab investigation (Lesson 2.1.2), you discovered how to make a parabola “sit” on the  $x$ -axis (the polynomial has one root), and you looked at ways of making parabolas intersect the  $x$ -axis in two specific places (two roots).



**8-36.** Where does the graph  $y = (x + 3)^2 - 5$  cross the  $x$ -axis? [Homework Help](#)

**8-37.** If you were to graph the function  $f(x) = (x - 74)^2(x + 29)$ , where would the graph intersect the  $x$ -axis? [Homework Help](#)

**8-38.** For each pair of intercepts given below, write an equation for a quadratic function in standard form. [Homework Help](#)

a.  $(-3, 0)$  and  $(2, 0)$

b.  $(-3, 0)$  and  $(\frac{1}{2}, 0)$

**8-39.** What is the degree of each polynomial function below? [Homework Help](#)

a.  $P(x) = 0.08x^2 + 28x$

b.  $y = 8x^2 - \frac{1}{7}x^5 + 9$

c.  $f(x) = 5(x + 3)(x - 2)(x + 7)$

d.  $y = (x - 3)^2(x + 1)(x^3 + 1)$

**8-40.** Consider the following types of graphs. Which graphs are polynomial functions? Explain your reasoning. [Homework Help](#)

- a. Parabolas

- b. Exponentials
- c. Cubics
- d. Lines
- e. Circles

**8-41.** Graph each system below and shade the solution region. [Homework Help](#) 

a.  $y \geq x^2 - 4$   
 $y < -3x + 1$

b.  $y < 2x + 5$   
 $y \geq |x + 1|$

**8-42.** A circle with its center on the line  $y = 3x$  in the 1<sup>st</sup> quadrant is tangent to the  $y$ -axis. [Homework Help](#) 

- a. If the radius is 2, what is the equation of the circle?
- b. If the radius is 3, what is the equation of the circle?

**8-43.** Sketch the graph of each function below on the same set of axes. [Homework Help](#) 

a.  $y = 2^x$   
b.  $y = 2^x + 5$   
c.  $y = 2^x - 5$

**8-44.** For each equation, find two solutions  $0^\circ \leq \theta < 360^\circ$ , which make the equation true. You should not need a calculator. [Homework Help](#) 

a.  $\sin \theta = \frac{1}{2}$   
b.  $\tan \theta = \sqrt{3}$   
c.  $\cos \theta = \frac{\sqrt{3}}{2}$   
d.  $\sin \theta = -\frac{\sqrt{2}}{2}$

