8.2.2 What are complex roots?

Complex Roots

In this lesson, you will solve equations as well as reverse your thinking to investigate the relationship between the complex solutions to a quadratic equation and the equation from which these solutions came.

8-79. Find the roots of each of the following quadratic functions by solving for x when $y = 0$. Does the graph of either of these functions intersect the x-axis?

a. $y = (x + 5)^2 + 9$

b. $y = x^2 - 4x + 9$

8-80. What do you notice about the complex solutions in problem 8-79? Describe any patterns you see. Discuss these with your team and write all of your observations.

8-81. In parts (a) through (d) below, look for patterns as you calculate the sum and the product for each pair of complex numbers. Use what you find to answer parts (e) through (g).

a. $2 + i, 2 - i$

b. $3 - 5i, 3 + 5i$

c. $-4 + i, -4 - i$

d. $1 + i\sqrt{3}, 1 - i\sqrt{3}$

e. What complex number can you multiply $3 + 2i$by to get a real number?

f. What happens when you multiply $(-4 + 5i)(-4 + 3i)$?

g. What complex number can you multiply $a + bi$by to get a real number?

8-82. WHAT EQUATION HAS THESE SOLUTIONS?

Each of the four pairs of complex numbers in problem 8-81 could be the roots of a quadratic function.

Your Task: With your team, create a quadratic equation for each pair of complex numbers in parts (a) through (d) of problem 8-81 such that those numbers are the roots. Discuss the methods you use for writing the equations and write summary statements describing your methods.

Discussion Points

How can we reverse the process of solving?

How can we use what we know about factors and zeros?

How are the solutions related to the standard form of the equation?

Further Guidance

8-83. Problem 8-81 made Mariposa curious about sums and products. She decided to solve the equation $x^2 - 6x + 25 = 0$ and look at the sums and products of its solutions. What patterns can you help her find that might give her ideas about the equation once she knows the solutions?

8-84. Austin had another idea. He knew that if 3 and −5 were solutions of a quadratic equation then $(x - 3)$ and $(x + 5)$ would be factors that could be multiplied to get a quadratic polynomial. How could his idea be used with the pairs of complex solutions in problem 8-81? Choose one pair and show how to use your idea.

8-85. Melvin had still another idea. "Why not just let $x = -4 \pm i$ and work backwards?" He asked. Would his idea work?

Further Guidance section ends here.

8-86. For each pair of numbers below, find a quadratic equation that has these numbers as solutions.

a. $\frac{3}{4}$ and -5

b. $3i$ and $-3i$

c. $5 + 2i$ and $5 - 2i$

d. $-3+\sqrt{2}$ and $-3-\sqrt{2}$

METHODS AND MEANINGS

MATH NOTES

The Discriminant and Complex Conjugates

With the introduction of complex numbers, the use of the terms **roots** and **zeros** of polynomials expands to include complex numbers that are solutions of the equations when $p(x) = 0$.

For any quadratic equation $ax^2 + bx + c = 0$, you can determine whether the roots are real or complex by examining the part of the quadratic formula that is under the square-root sign. The value of $b^2 - 4ac$ is known as the **discriminant**. The roots are real when $b^2 - 4ac \geq 0$ and complex when $b^2 - 4ac < 0$.

For example, in the equation $2x^2 - 3x + 5 = 0$, $b^2 - 4ac = (-3)^2 - 4(2)(5) = -31 < 0$, so the equation has two complex roots and the parabola $y = 2x^2 - 3x + 5$ does not intersect the x-axis.

Complex roots of quadratic equations with real coefficients will have the form $a - bi$ and $a + bi$, which are called **complex conjugates**. The sum and product of two complex conjugates are always real numbers.

For example, the conjugate for the complex number $-5 + 3i$, is $-5 - 3i$. $(-5 + 3i) + (-5 - 3i) = -10$ and $(-5 + 3i)(-5 - 3i) = 25 - 9i^2 = 34$.

Review & Preview

8-87. For each of the following sets of numbers, find the equation of a function that has these numbers as roots. Homework Help ✎

a. $-3 + i$ and $-3 - i$

b. $5+\sqrt{3}$ and $5-\sqrt{3}$

c. -2, $\sqrt{7}$, and $-\sqrt{7}$

d. 4, $-3 + i$, and $-3 - i$

8-88. Raul claims that he has a shortcut for deciding what kind of roots a function has. Jolene thinks that a shortcut is not possible. She says you just have to solve the quadratic equation to find out. They are working on $y = x^2 - 5x - 14$.

Jolene says, "*See, I just start out by trying to factor. This one can be factored* $(x - 7)(x + 2) = 0$, *so the equation will have two real solutions and the function will have two real roots.*"

"*But what if it can't be factored*?" Raul asked. "*What about* $x^2 + 2x + 2 = 0$?"

"*That's easy! I just use the Quadratic Formula,*" says Jolene. "*And I get... let's see... negative two plus or minus the square root of... two squared... that's 4... minus... eight...*"

"*Wait!*" Raul interrupted. "*Right there, see, you don't have to finish.* 2^2 *minus* $4 \cdot 2$, *that gives you* -4. *That's all you need to know. You'll be taking the square root of a negative number so you will get a complex result.*"

> *"Oh, I see,"* said Jolene. *"I only have to consider part of the solution, the inside of the square root."*

Use Raul's method to tell whether each of the following functions has real or complex roots without completely solving the equation. Note: Raul's method is also summarized in the Math Notes box for this lesson. Homework Help ✎

a. $y = 2x^2 + 5x + 4$

b. $y = 2x^2 + 5x - 3$

8-89. Sketch the graphs and find the area of the intersection of the inequalities below. Homework Help ✎

$$y > |x+3|$$
$$y \leq 5$$

8-90. Consider this geometric sequence: $i^0, i^1, i^2, i^3, i^4, i^5, \ldots, i^{15}$. Homework Help ✎

a. You know that $i^0 = 1$, $i^1 = i$, and $i^2 = -1$. Calculate the result for each term up to i^{15}, and describe the pattern.

b. Use the pattern you found in part (a) to calculate i^{16}, i^{25}, i^{39}, and i^{100}.

c. What is i^{4n}, where n is a positive whole number?

d. Based on your answer to part (c), simplify i^{4n+1}, i^{4n+2}, and i^{4n+3}.

e. Calculate i^{396}, i^{397}, i^{398}, and i^{399}.

8-91. Use the pattern from the previous problem to help you to evaluate the following expressions. Homework Help ✎

a. i^{592}

b. i^{797}

c. $i^{10,648,202}$

8-92. Describe how you would evaluate i^n where n could be any integer. Homework Help ✎

8-93. Show how to solve the equations below *without* using your calculator. You will have radicals or logarithms in your answers. Homework Help ✎

a. $3^x = 17$

b. $x^3 = 17$

8-94. Match each equation with the appropriate graph. Do this without using a graphing calculator. Homework Help ✎

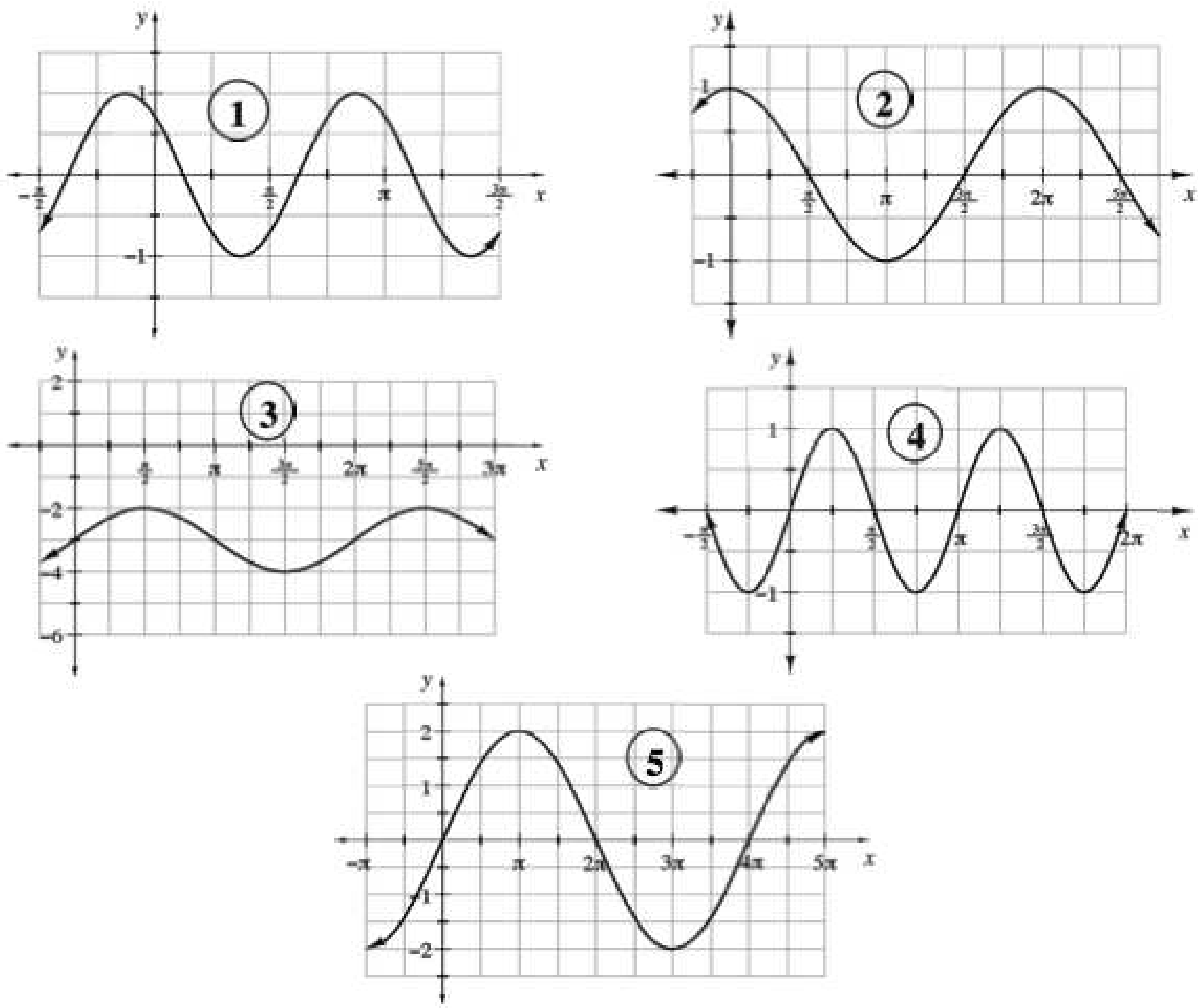
a. $y = \sin(x + \frac{\pi}{2})$

b. $y = \sin(2x)$

c. $y = 2\sin(\frac{x}{2})$

d. $y = \sin(x) - 3$

e. $y = -\sin[2(x - \frac{\pi}{8})]$

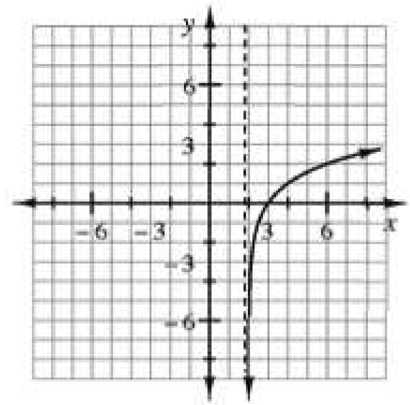


8-95. The La Quebrada Cliff Divers perform shows for the public by jumping into the sea off the cliffs at Acapulco, Mexico. The height (in feet) of a diver at a certain time (in seconds) is given by $h = -16t^2 + 16t + 400$. Homework Help ✎

a. Use the vertex and y-intercept to make a sketch that represents the dive. What form of the quadratic function helps you determine the y-intercept efficiently? What form helps you determine the vertex easily?

b. At what height did the diver start his jump? What is the maximum height he achieved?

8-96. Consider the graph at right. Homework Help ✎



a. What is the parent for this function?

b. What is the equation of the vertical asymptote?

c. Write a possible equation for this graph.