8.3.1 How can I divide polynomials?

Polynomial Division

When you graphed polynomial functions in the first section of this chapter, you learned that the factored form of a polynomial is very useful for finding the roots of the function or the x-intercepts of the graph. But what happens when you do not have the factored form and you need to find all of the roots? You will investigate the answers to this question in this lesson.

8-113. Andre needs to find the exact roots of the function $f(x) = x^3 + 2x^2 - 7x - 2$. When he uses his graphing calculator, he can see that one of the x-intercepts is 2, but there are two other intercepts that he cannot identify exactly.

Andre remembers that he learned how to multiply binomials and other polynomials using area models. He figures that since division is the inverse (or undo) operation for multiplication, he should be able to reverse the multiplication process to divide. As he thinks about that idea, he comes across the following news article.

Polydoku Craze Sweeping Nation!

(CPM) - Math enthusiasts around the nation have entered a new puzzle craze involving the multiplication of polynomials. The goal of the game, which enthusiasts have named Polydoku, is to fill in squares so that the multiplication of two polynomials will be completed.

The game shown right, for example, represents the multiplication of $(3x - 2)(2x^3 - x^2 + 3x - 1) = 6x^4 - 7x^3 + 11x^2 - 9x + 2$.

	1	2	3	4	5
A	×	$2x^3$	$-x^2$	$+3x$	-1
B	$3x$	$6x^4$	$-3x^3$	$9x^2$	$-3x$
C	-2	$-4x^3$	$2x^2$	$-6x$	2
		$6x^4$	$-7x^3$	$+11x^2$	$-9x$	$+2$

Most of the squares are blank at the start of the game. While the beginner level provides the factors (in the gray squares), some of the factors are missing in the more advanced levels.

What do you think Andre needs to be able to do to find the other roots?

8-114. Andre decided to join the craze and try some Polydoku puzzles, but he is not sure how to fill in some of the squares. Help him by answering parts (a) and (b) below about the Polydoku puzzle in the news article he read (found in problem 8-113), then complete part (c).

a. Explain how the term $2x^2$ in cell C3 of the news article was generated.

b. What values were combined to get $-7x^3$ in the news article answer?

c. Copy and complete the Polydoku puzzle below.

	1	2	3	4	5
A	×	$4x^3$	$+6x^2$	$-2x$	-5
B	$2x$				
C	-3				

8-115. POLYDOKU TEAM CHALLENGE

Work with your team to complete the puzzle below. Find the factors and the product for the puzzle. If you get stuck, you can consult parts (a) through (c) below for ideas.

	1	2	3	4	5
A	×			$-2x$	
B	x	$2x^4$			
C	-4		$12x^2$		

$12x$

a. How is cell B2 related to the answer?

b. How did you find the third term in the answer?

c. What cells did you use to get the value in cell B5?

8-116. Jessica is about to start the intermediate-level Polydoku puzzle shown below. Show Jessica how to complete the puzzle. Make sure you can justify your solution.

Use your results to complete the statements below.

	1	2	3	4
A	×			
B	$2x$			
C	$+5$			

$6x^3$ $+7x^2$ $-16x$ $+10$

$$\frac{6x^3+7x^2-16x+10}{2x+5}=\_\_\_\_\_\_\_\_\_\_ \text{ and } (2x+5)\cdot\_\_\_\_\_\_\_\_\_\_=\_\_\_\_\_\_\_\_\_\_\_\_$$

8-117. Unfortunately, Jessica made a mistake when she copied the problem. The constant term of the original polynomial was supposed to have the value + 18 (not + 10). She does not want to start all over again to solve the puzzle.

a. Jessica realizes that she now has 8 remaining from the original expression. What is the significance of this 8?

b. Jessica writes her work as shown below:

$$\frac{6x^3+7x^2-16x+18}{2x+5} = \frac{(6x^3+7x^2-16x+10)+8}{2x+5} = 3x^2-4x+2 \text{, remainder } 8.$$

Gina thinks that there is a way to write the answer without using the word “remainder.” Discuss this with your team and find another way to write the result. Be prepared to share your results and your reasoning with the class.

c. Use Jessica and Gina's method to divide $(6x^3 + 11x^2 - 12x - 1) \div (3x + 1)$.

8-118. Create your own Polydoku puzzles that can be used to solve each of the polynomial-division problems below. Express any remainders as fractions and use your results to write a multiplication and a division statement such as those in problem 8-117.

a. $\frac{6x^4-5x^3+10x^2-18x+5}{3x-1}$

b. $(x^4-6x^3+18x-4)\div(x-2)$

c. $x-3\overline{)x^3+x^2-14x+3}$

d. $\frac{x^5-1}{x-1}$

8-119. Now work with your team to help Andre solve his original problem (problem 8-113). Find all of the roots (exact zeros) of the polynomial.

METHODS AND MEANINGS

MATH NOTES

Polynomial Division

The examples below show two methods for dividing $x^4 - 6x^3 + 18x - 1$ by $x - 2$. In both cases, the remainder is written as a fraction.

Using long division:

Using Area Models:

$$
\begin{array}{r}
x^3-4x^2-8x+2 \\
x-2\overline{)x^4-6x^3+0x^2+18x-1} \\
\underline{x^4-2x^3} \\
-4x^3+0x^2 \\
\underline{-4x^3+8x^2} \\
-8x^2+18x \\
\underline{-8x^2+16x} \\
2x-1 \\
\underline{2x-4} \\
3
\end{array}
$$

Answer: $x^3-4x^2-8x+2+\frac{3}{x-2}$

	x^3	$-4x^2$	$-8x$	$+2$	
x	x^4	$-4x^3$	$-8x^2$	$+2x$	3
-2	$-2x^3$	$+8x^2$	$+16x$	-4	

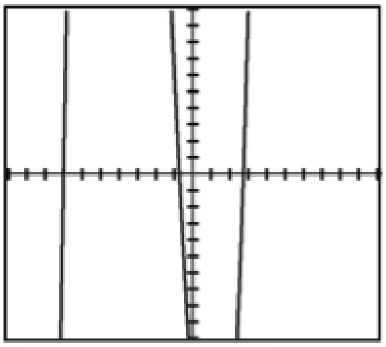
$x^4-6x^3 \quad +0x^2 \quad +18x \quad -1$

Answer: $x^3-4x^2-8x+2+\frac{3}{x-2}$

Therefore, $(x^4-6x^3+18x-1)\div(x-2)=x^3-4x^2-8x+2+\frac{3}{x-2}$ and $(x-2)(x^3-4x^2-8x+2+\frac{3}{x-2})=x^4-6x^3+18x-1$

Review & Preview

8-120. Carlos is always playing games with his graphing calculator, but now his calculator has contracted a virus. The TRACE, ZOOM, and WINDOW functions on his calculator are not working. He needs to solve $x^3+5x^2-16x-14=0$, so he graphs $y=x^3+5x^2-16x-14$ and sees the graph at right in the standard window. Homework Help ✎



a. From the graph, what appears to be an integer solution to the equation?

b. Check your answer from part (a) in the equation.

c. Since $x=-7$ is a solution to the equation, what is the factor associated with this solution?

d. Use polynomial division to find the other factor.

e. Use your new factor to complete this equation:
$x^3+5x^2-16x-14=(x+7)(\textit{other factor})=0$

f. The "other factor" leads to two other solutions to the equation. Find these two new solutions and give all three solutions to the original equation.

8-121. Now Carlos needs to solve $2x^3+3x^2-8x+3=0$, but his calculator will still only create a standard graph. He sees that the graph of $y=2x^3+3x^2-8x+3$ crosses the x-axis at $x=1$. Find all three solutions to the equation. Homework Help ✎

8-122. Without actually multiplying, decide which of the following polynomials could be the product of $(x-2)(x+3)(x-5)$. Justify your choice. Homework Help ✎

a. $x^3 - 4x^2 - 11x - 5$

b. $2x^3 - 4x^2 - 11x + 30$

c. $x^3 - 4x^2 - 11x + 30$

d. $2x^3 - 4x^2 - 11x - 5$

8-123. Which of the following binomials could be a factor of $x^3 - 9x^2 + 19x + 5$? Explain your reasoning. Homework Help ✎

a. $x - 2$

b. $x - 5$

c. $x + 3$

d. $x + 2$

8-124. Now divide $x^3 - 9x^2 + 19x + 5$ by the factor that you chose in the preceding problem. If it is a factor, use it and the resulting factor to find all the zeros of the polynomial. If it is not a factor, reconsider your answer to the preceding problem and try a different factor. Homework Help ✎

8-125. Consider the equation $5x^2 - 7x - 6 = 0$ as you answer the questions in parts (a) through (d) below. Homework Help ✎

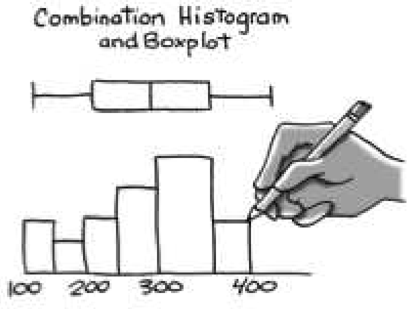
a. What are the factors of $5x^2 - 7x - 6$?

b. What are the solutions to the equation?

c. Explain the relationship between the factors of the polynomial expression and the solutions to the equation.

d. How are the solutions to the equation related to the lead coefficient and constant term in the original polynomial?

8-126. The city of Waynesboro is trying to decide whether to initiate a composting project where each residence would be provided with a dumpster for garden and yard waste. The city manager needs some measure of assurance that the citizens will participate before launching the project, so he chooses a random sample of 25 homes and provides them with the new dumpster for yard and garden waste. After one week the contents of each dumpster is weighed (in pounds) before processing. The sorted data is shown below: Homework Help ✎



0	0	0	0	1.7	2.6	2.9	4.2	4.4
5.1	5.6	6.4	8.0	8.9	9.7	10.1	11.2	13.5
15.1	16.3	17.7	21.4	22.0	22.2	36.5		

Checksum 245.5

a. Create a combination boxplot and histogram. Use an interval of 0 to 42 pounds on the x-axis and a bin width of 6 pounds.

b. Describe the center, shape, spread and outliers.

c. What is a better measure of center for this distribution the mean or median and why?

d. What is a better measure of spread the standard deviation or IQR and why?

e. The city can sell the compost, and engineers estimate the program will be profitable if each home averages at least 9 pounds of material. The city manager sees the mean is nearly 10 pounds and is ready to order dumpsters for every residence. What advice would you give him?

8-127. This problem is a checkpoint for solving and graphing inequalities. It will be referred to as Checkpoint 8A. Homework Help ✎

Graph the inequality in part (a) and the system of inequalities in part (b).

a. $|x+1| \geq 3$

b. $y \leq -2x + 3$
$y \geq x$
$x \geq -1$

Check your answers by referring to the Checkpoint 8A materials.

Ideally, at this point you are comfortable working with these types of problems and can solve them correctly. If you feel that you need more confidence when solving these types of problems, then review theCheckpoint 8A materials and try the practice problems provided. From this point on, you will be expected to do problems like these correctly and with confidence.

8-128. Given the equation: $3x + y - z = 6$. Homework Help ✎

a. Draw a graph.

b. Is $(1, 2, -1)$ on the graph? Justify your answer.