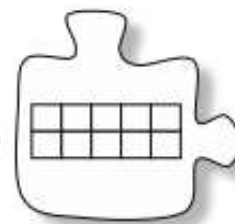


8.3.3 How can I use it?

An Application of Polynomials



In this lesson, you will have the opportunity to use the equation and graph of a polynomial function to solve a problem involving a game at the County Fair.

8-165. COUNTY FAIR GAME TANK

The Mathamericaland Carnival Company wants to create a new game. It will consist of a tank filled with Ping-Pong balls of different colors. People will pay for the opportunity to crawl around in the tank blindfolded for 60 seconds, while they collect Ping-Pong balls. Most of the Ping-Pong balls will be white, but there will be a few of different colors. The players will win \$100 for each red, \$200 for each blue, and \$500 for each green Ping-Pong ball they carry out of the tank.

The tank will be rectangular, open at the top, and will be made by cutting squares out of each corner of an 8.5-meter by 11-meter sheet of translucent miracle material that can be bent into shape. The owner of the company thinks that she will make the greatest profit if the tank has maximum volume. She has hired your team to figure out the exact dimensions of the tank that creates the maximum value.



Your Task: Your team will write a report of your findings to the Carnival Company that includes each of the following elements.

- Any data or conjectures your team made based on experimental paper tanks.
- A diagram of the tank with the dimensions clearly labeled with appropriate variables.
- A graph of the volume function that you found with notes on a reasonable domain and range.
- An equation that matches your graph.
- Your conclusions and observations.

Further Guidance

8-166. Use a full sheet of 8.5" x 11" paper, which is the same shape as the material for the tank. Each member of your team should choose a different sized square to cut out of the corners. Measure the side of the square you cut out and write along the edge of the large piece of paper. Use either inches or centimeters. Your paper should look like the figure at right.

Fold the paper up into an open box (fold on the dotted lines). Then tape the cut parts together so that the

box holds its shape. Measure the dimensions of the tank. Record the dimensions directly on the model tank.

8-167. Make a table like the one below for your team's results. Consider “extreme” tanks, the ones with the largest possible cutout and the smallest possible cutout. For example, imagine cutting a square out of each corner zero inches on a side.



Height of Tank	Width of Tank	Length of Tank	Volume of Tank

- Examine the data in the table with your team and make some conjectures about how to find the maximum volume.
- Label the height as x . Using x for the height, find expressions for the length and width.
- Write an equation to represent the volume of the tank.
- Sketch the graph of your function by using the roots and determining the orientation.
- What domain and range make sense for your function?
- Approximate the maximum volume of the tank and the dimensions of the tank that will generate this volume.

8-168. Use your graph and your tank model to write your report. Include your answers to the following questions.

- Which points on the graph represent tanks that can actually be made? Explain.
- How are the dimensions of the tank related? In other words, what happens to the length and width as the height increases?
- Make a drawing of your tank. (You may want to use isometric dot paper.) Label your drawing with its dimensions and its volume.

===== *Further Guidance* =====
section ends here.

METHODS AND MEANINGS

MATH NOTES

Factoring Sums and Differences

The difference of two squares can be factored: $a^2 - b^2 = (a + b)(a - b)$

The sum of two cubes can be factored: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

The difference of two cubes can be factored: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$



8-169. A polynomial function has the equation $P(x) = x(x - 3)^2(2x + 1)$. What are the x -intercepts?

[Homework Help](#)

8-170. Sketch a graph of a fourth-degree polynomial that has no real roots. [Homework Help](#)

8-171. Generally, when you are asked to factor, it is understood that you are only to use integers in your factors. If you are allowed to use irrational or complex numbers, any quadratic can be factored.

By setting the polynomial equal to zero and solving the quadratic equation, you can work backwards to “force factor” any quadratic. Use the solutions of the corresponding quadratic equation to write each of the following expressions as a product of two linear factors. [Homework Help](#)

a. $x^2 - 10$

b. $x^2 - 3x - 7$

c. $x^2 + 4$

d. $x^2 - 2x + 2$

8-172. Decide which of the following equations have real roots, and which have complex roots without completely solving them. [Homework Help](#)

a. $y = x^2 - 6$

b. $y = x^2 + 6$

c. $y = x^2 - 2x + 10$


d. $y = x^2 - 2x - 10$

e. $y = (x - 3)^2 - 4$

f. $y = (x - 3)^2 + 4$

8-173. Determine whether $x = -2$ is a solution to the equation $x^4 - 4x = 8x^2 - 40$. Show why or why not.

[Homework Help](#) 

8-174. This problem is a checkpoint for solving complicated equations. It will be referred to as Checkpoint 8B. [Homework Help](#) 

Solve each equation. Check your solutions.

a. $2|x - 3| + 7 = 11$


b. $4(x - 2)^2 = 16$

c. $\sqrt{x + 18} = x - 2$

d. $|2x + 5| = 3x + 4$


Check your answers by referring to the [Checkpoint 8B materials](#).

Ideally, at this point you are comfortable working with these types of problems and can solve them correctly. If you feel that you need more confidence when solving these types of problems, then review the [Checkpoint 8B materials](#) and try the practice problems provided. From this point on, you will be expected to do problems like these correctly and with confidence.

8-175. Let $p(x) = x^3 - 3x^2 - 7x + 9$. [Homework Help](#) 

a. Find $p(5)$.

b. Verify the Remainder Theorem. Read about the Remainder Theorem in the Math Notes box in Lesson 8.3.2.

8-176. The roots of two quadratic polynomials are given below. Write possible quadratic functions in standard form. [Homework Help](#) 

a. $x = -i, x = i$

b. $x = 1 + \sqrt{2}, x = 1 - \sqrt{2}$

8-177. Graph two cycles of each function. [Homework Help](#) 

a. $y = -2\cos(x + \frac{\pi}{2})$

b. $y = \sin(x - \frac{\pi}{2})$