Here are some more to try. Factor each expression.

1. $2x^2 + 7x - 4$
2. $7x^2 + 13x - 2$
3. $3x^2 + 11x + 10$
4. $x^2 + 5x - 24$
5. $2x^2 + 5x - 7$
6. $3x^2 - 13x + 4$
7. $64x^2 + 16x + 1$
8. $5x^2 + 12x - 9$
9. $8x^2 + 24x + 10$
10. $6x^3 + 31x^2 + 5x$

**Answers:**

1. $(x+4)(2x-1)$
2. $(7x-1)(x+2)$
3. $(3x+5)(x+2)$
4. $(x+8)(x-3)$
5. $(2x+7)(x-1)$
6. $(3x-1)(x-4)$
7. $(8x+1)^2$
8. $(5x-3)(x+3)$
9. $2(4x^2+12x+5) = 2(2x+1)(2x+5)$
10. $x(6x^2+31x+5) = x(6x+1)(x+5)$

# Checkpoint 6A

## Problem 6-73

### Multiplying and Dividing Rational Expressions

Answers to problem 6-73: a: $\frac{x+3}{x+4}$ , b: $\frac{1}{x(x+2)}$

Multiplication or division of rational expressions follows the same procedure used with numerical fractions. However, it is often necessary to factor the polynomials in order to simplify them. Factors that are the same in the numerator and denominator are equal to 1. For example: $\frac{x^2}{x^2}=1$, $\frac{(x+2)}{(x+2)}=1$ and $\frac{(3x-2)}{(3x-2)}=1$ but $\frac{5-x}{x-5}=\frac{-(x-5)}{x-5}=-1$.

When dividing rational expressions, change the problem to multiplication by inverting (flipping) the second expression (or any expression that follows a division sign) and completing the process as you do for multiplication.

In both cases, the simplification is only valid provided that the denominator is not equal to zero. See the examples below.

**Example 1: Multiply $\frac{x^2+6x}{(x+6)^2}\cdot\frac{x^2+7x+6}{x^2-1}$ and simplify the result.**

Solution:

After factoring, the expression becomes: $\frac{x(x+6)}{(x+6)(x+6)}\cdot\frac{(x+6)(x+1)}{(x+1)(x-1)}$

After multiplying, reorder the factors: $\frac{(x+6)}{(x+6)}\cdot\frac{(x+6)}{(x+6)}\cdot\frac{x}{(x-1)}\cdot\frac{(x+1)}{(x+1)}$

Since $\frac{(x+6)}{(x+6)}=1$ and $\frac{(x+1)}{(x+1)}=1$, simplify: $1\cdot1\cdot\frac{x}{x-1}\cdot1 \Rightarrow \frac{x}{x-1}$ for $x\neq-6,-1,$ or $1$.

**Example 2: Divide $\frac{x^2-4x-5}{x^2-4x+4}\div\frac{x^2-2x-15}{x^2+4x-12}$ and simplify the result.**

Solution:

First change to a multiplication expression by inverting (flipping) the second fraction: $\frac{x^2-4x-5}{x^2-4x+4}\cdot\frac{x^2+4x-12}{x^2-2x-15}$

After factoring, the expression is: $\frac{(x-5)(x+1)}{(x-2)(x-2)}\cdot\frac{(x+6)(x-2)}{(x-5)(x+3)}$

Reorder the factors (if you need to): $\frac{(x-5)}{(x-5)}\cdot\frac{(x-2)}{(x-2)}\cdot\frac{(x+1)}{(x-2)}\cdot\frac{(x+6)}{(x+3)}$

Since $\frac{(x-5)}{(x-5)}=1$ and $\frac{(x-2)}{(x-2)}=1$, simplify: $\frac{(x+1)}{(x-2)}\cdot\frac{(x+6)}{(x+3)}$

Thus, $\frac{x^2-4x-5}{x^2-4x+4}\div\frac{x^2-2x-15}{x^2+4x-12}=\frac{(x+1)}{(x-2)}\cdot\frac{(x+6)}{(x+3)}$ or $\frac{x^2+7x+6}{x^2+x-6}$ for $x\neq-3,$ $2,$ or $5$.

Now we can go back and solve the original problems.

a. $$\frac{x^2-16}{(x-4)^2}\cdot\frac{x^2-3x-18}{x^2-2x-24}\Rightarrow\frac{(x+4)(x-4)}{(x-4)(x-4)}\cdot\frac{(x-6)(x+3)}{(x-6)(x+4)}\Rightarrow\frac{(x+4)(x-4)(x-6)(x+3)}{(x+4)(x-4)(x-6)(x-4)}\Rightarrow\frac{x+3}{x-4}$$

b. $$\frac{x^2-1}{x^2-6x-7}\div\frac{x^3+x^2-2x}{x-7}\Rightarrow\frac{(x+1)(x-1)}{(x-7)(x+1)}\cdot\frac{(x-7)}{x(x+2)(x-1)}\Rightarrow\frac{(x+1)(x-1)(x-7)}{(x+1)(x-1)(x-7)x(x+2)}\Rightarrow\frac{1}{x(x+2)}$$

Here are some more to try. Multiply or divide each pair of rational expressions. Simplify the result. Assume the denominator is not equal to zero.

1. $\frac{x^2+5x+6}{x^2-4x}\cdot\frac{4x}{x+2}$
2. $\frac{x^2-2x}{x^2-4x+4}\div\frac{4x^2}{x-2}$
3. $\frac{x^2-16}{(x-4)^2}\cdot\frac{x^2-3x-18}{x^2-2x-24}$
4. $\frac{x^2-x-6}{x^2+3x-10}\cdot\frac{x^2+2x-15}{x^2-6x+9}$
5. $\frac{x^2-x-6}{x^2-x-20}\cdot\frac{x^2+6x+8}{x^2-x-6}$
6. $\frac{x^2-x-30}{x^2+13x+40}\cdot\frac{x^2+11x+24}{x^2-9x+18}$
7. $\frac{15-5x}{x^2-x-6}\div\frac{5x}{x^2+6x+8}$
8. $\frac{17x+119}{x^2+5x-14}\div\frac{9x-1}{x^2-3x+2}$
9. $\frac{2x^2-5x-3}{3x^2-10x+3}\cdot\frac{9x^2-1}{4x^2+4x+1}$
10. $\frac{x^2-1}{x^2-6x-7}\div\frac{x^3+x^2-2x}{x-7}$
11. $\frac{3x-21}{x^2-49}\div\frac{3x}{x^2+7x}$
12. $\frac{x^2-y^2}{x+y}\cdot\frac{1}{x-y}$
13. $\frac{y^2-y}{w^2-y^2}\div\frac{y^2-2y+1}{1-y}$
14. $\frac{y^2-y-12}{y+2}\div\frac{y-4}{y^2-4y-12}$
15. $\frac{x^2+7x+10}{x+2}\div\frac{x^2+2x-15}{x+2}$

**Answers:**

1. $\frac{4(x+3)}{x-4}$
2. $\frac{1}{4x}$
3. $\frac{x+3}{x-4}$
4. $\frac{x+2}{x-2}$
5. $\frac{x+2}{x-5}$
6. $\frac{x+3}{x-3}$
7. $\frac{-x-4}{x}$
8. $\frac{17(x-1)}{9x-1}$
9. $\frac{3x+1}{2x+1}$
10. $\frac{1}{x(x+2)}$
11. 1
12. 1
13. $\frac{-y}{w^2-y^2}$
14. $(y+3)(y-6)$
15. $\frac{x+2}{x-3}$

# Checkpoint 6B

## Problem 6-145

### Adding and Subtracting Rational Expressions

Answers to problem 6-145: a: $\frac{2(x+1)}{x+3}$, b: $\frac{3x^2-5x-3}{(2x+1)^2}$

Addition and subtraction of rational expressions uses the same process as simple numerical fractions. First, if necessary find a common denominator. Second, convert the original fractions to equivalent ones with the common denominator. Third, add or subtract the new numerators over the common denominator. Finally, factor the numerator and denominator and simplify, if possible. Note that these steps are only valid provided that the denominator is not zero.

**Example 1:** $\frac{3}{2(n+2)}+\frac{3}{n(n+2)}$

Solution:
The least common multiple of $2(n+2)$ and $n(n+2)$ is $2n(n+2)$.

$$\frac{3}{2(n+2)}+\frac{3}{n(n+2)}$$

To get a common denominator in the first fraction, multiply the fraction by $\frac{n}{n}$, a form of the number 1. Multiply the second fraction by $\frac{2}{2}$.

$$=\frac{3}{2(n+2)}\cdot\frac{n}{n}+\frac{3}{n(n+2)}\cdot\frac{2}{2}$$

Multiply the numerator and denominator of each term.
It may be necessary to distribute the numerator.

$$=\frac{3n}{2n(n+2)}+\frac{6}{2n(n+2)}$$

Add, factor, and simplify the result. (Note: $n \neq 0$ *or* $-2$)

$$=\frac{3n+6}{2n(n+2)} \Rightarrow \frac{3(n+2)}{2n(n+2)} \Rightarrow \frac{3}{2n}$$

**Example 2:** $\frac{2-x}{x+4}+\frac{3x+6}{x+4}$

Solution:

$$\frac{2-x}{x+4}+\frac{3x+6}{x+4} \Rightarrow \frac{2-x+3x+6}{x+4} \Rightarrow \frac{2x+8}{x+4} \Rightarrow \frac{2(x+4)}{x+4} \Rightarrow 2$$

**Example 3:** $\frac{3}{x-1}-\frac{2}{x-2}$

Solution:

$$\frac{3}{x-1}-\frac{2}{x-2} \Rightarrow \frac{3}{x-1}\cdot\frac{x-2}{x-2}-\frac{2}{x-2}\cdot\frac{x-1}{x-1} \Rightarrow \frac{3x-6-2x+2}{(x-1)(x-2)} \Rightarrow \frac{x-4}{(x-1)(x-2)}$$

Now we can go back and solve the original problems.

a. $$\frac{4}{x^2+5x+6}+\frac{2x}{x+2} \Rightarrow \frac{4}{(x+3)(x+2)}+\frac{2x}{(x+2)}\cdot\frac{(x+3)}{(x+3)} \Rightarrow \frac{2x^2+6x+4}{(x+2)(x+3)} \Rightarrow \frac{2(x+2)(x+1)}{(x+2)(x+3)} \Rightarrow \frac{2(x+1)}{(x+3)}$$

b. $$\frac{3x^2+x}{(2x+1)^2}-\frac{3}{(2x+1)} \Rightarrow \frac{3x^2+x}{(2x+1)^2}-\frac{3}{(2x+1)}\cdot\frac{(2x+1)}{(2x+1)} \Rightarrow \frac{3x^2+x-6x-3}{(2x+1)^2} \Rightarrow \frac{3x^2-5x-3}{(2x+1)^2}$$

Here are a few more to try. Add or subtract each expression and simplify the result. In each case assume the denominator does not equal zero.

1. $\frac{x}{(x+2)(x+3)}+\frac{2}{(x+2)(x+3)}$

2. $\frac{x}{x^2+6x+8}+\frac{4}{x^2+6x+8}$

3. $\frac{b^2}{b^2+2b-3}+\frac{-9}{b^2+2b-3}$

4. $\frac{2a}{a^2+2a+1}+\frac{2}{a^2+2a+1}$

5. $\frac{x+10}{x+2}+\frac{x-6}{x+2}$

6. $\frac{a+2b}{a+b}+\frac{2a+b}{a+b}$

7. $\frac{3x-4}{3x+3}-\frac{2x-5}{3x+3}$

8. $\frac{3x}{4x-12}-\frac{9}{4x-12}$

9. $\frac{6a}{5a^2+a}-\frac{a-1}{5a^2+a}$

10. $\frac{x^2+3x-5}{10}-\frac{x^2-2x+10}{10}$

11. $\frac{6}{x(x+3)}+\frac{2x}{x(x+3)}$

12. $\frac{5}{x-7}+\frac{3}{4(x-7)}$

13. $\frac{5x+6}{x^2}-\frac{5}{x}$

14. $\frac{2}{x+4}-\frac{x-4}{x^2-16}$

15. $\frac{10a}{a^2+6a}-\frac{3}{3a+18}$

16. $\frac{3x}{2x^2-8x}+\frac{2}{x-4}$

17. $\frac{5x+9}{x^2-2x-3}+\frac{6}{x^2-7x+12}$

18 $\frac{x+4}{x^2-3x-28}-\frac{x-5}{x^2+2x-35}$

19. $\frac{3x+1}{x^2-16}-\frac{3x+5}{x^2+8x+16}$

20. $\frac{7x-1}{x^2-2x-3}-\frac{6x}{x^2-x-2}$

**Answers:**

1. $\frac{1}{x+3}$
2. $\frac{1}{x+2}$
3. $\frac{b-3}{b-1}$
4. $\frac{2}{a+1}$
5. 2
6. 3
7. $\frac{1}{3}$
8. $\frac{3}{4}$
9. $\frac{1}{a}$
10. $\frac{x-3}{2}$
11. $\frac{2}{x}$
12. $\frac{23}{4(x-7)}=\frac{23}{4x-28}$
13. $\frac{6}{x^2}$
14. $\frac{1}{x+4}$
15. $\frac{9}{a+6}$
16. $\frac{7}{2(x-4)}=\frac{7}{2x-8}$
17. $\frac{5(x+2)}{(x-4)(x+1)}=\frac{5x+10}{x^2-3x-4}$
18. $\frac{14}{(x-7)(x+7)}=\frac{14}{x^2-49}$
19. $\frac{4(5x+6)}{(x-4)(x+4)^2}$
20. $\frac{x+2}{(x-3)(x-2)}=\frac{x+2}{x^2-5x+6}$

# Checkpoint 7A

## Problem 7-67

### Finding the $x$- and $y$-Intercepts of a Quadratic Function

Answers to problem 7-67: $y$-intercept: $(0,-17)$, $x$-intercepts: $(-2+\sqrt{21},0)$ and $(-2-\sqrt{21},0)$.

The $y$-intercepts of an equation are the points at which the graph crosses the $y$-axis. To find the $y$-intercept of an equation, substitute $x=0$ into the equation and solve for $y$.

The $x$-intercepts of an equation are the points at which the graph crosses the $x$-axis. To find the $x$-intercepts of an equation, substitute $y=0$ into the equation and solve for $x$. For a quadratic, you can do this by factoring and using the Zero Product Property or by using the Quadratic Formula, as well as other methods.

**Example 1: Find the $x$-intercepts of the graph of the equation $y=x^2+4x-12$.**

Solution: If $y=0$, then: $0=x^2+4x-12$

By factoring and using the Zero Product Property: $0=(x+6)(x-2)$

$x+6=0$ or $x-2=0$

$x=-6$ or $x=2$

Answers: The $x$-intercepts are $(-6, 0)$ and $(2, 0)$.

**Example 2: Find the $x$-intercepts of the graph of the equation $y=2x^2-3x-3$.**

Solution: If $y=0$, then: $0=2x^2-3x-3$

Since we cannot factor the trinomial we use the Quadratic Formula to solve for $x$.

If $ax^2+bx+c=0$ then: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Substitute $a=2$, $b=-3$, $c=-3$. $x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-3)}}{2(2)}$

Simplify. $x=\frac{3\pm\sqrt{9+24}}{4}=\frac{3\pm\sqrt{33}}{4}$

Find $\sqrt{33}$ approximately: $\approx\frac{3\pm 5.745}{4}$, so $\frac{3+5.745}{4}$ and $\frac{3-5.745}{4}$.

Answers: Simplify the fractions and the $x$-intercepts are approximately $(2.19,0)$ and $(-0.69,0)$. They can be expressed in exact form as $\left(\frac{3+\sqrt{33}}{4},0\right)$ and $\left(\frac{3-\sqrt{33}}{4},0\right)$.

Now we can find the $x$- and $y$-intercepts in the original problem.

Find the $x$ and $y$ intercepts of the graph of $y = x^2 + 4x - 17$.

Solution: To find the $y$-intercept, let $x = 0$ so $y = (0)^2 + 4(0) - 17 = -17$.
To find the $x$-intercept, let $y = 0$ so $0 = x^2 + 4x - 17$.
Since we cannot factor we use the Quadratic Formula with $a = 1$, $b = 4$ and $c = -17$.

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-17)}}{2(1)} = \frac{-4 \pm \sqrt{16+68}}{2} = \frac{-4 \pm \sqrt{84}}{2} = \frac{-4 \pm 2\sqrt{21}}{2} = -2 \pm \sqrt{21}$$

Answers: The $y$-intercept is $(0, -17)$. The $x$-intercepts are $(-2 \pm \sqrt{21}, 0)$, or approximately $(2.58, 0)$ and $(-6.58, 0)$.

Here are some more to try. Find the $x$- and $y$-intercepts for the graphs of each equation.

1. $y = x^2 + 2x - 15$
2. $y = 2x^2 + 7x + 3$
3. $y = 3x^2 - 5x + 2$
4. $y = 4x^2 - 8x$
5. $y = 2x^2 - 9x - 35$
6. $y = 2x^2 - 11x + 5$
7. $3x^2 + 2 + 7x = y$
8. $8x^2 + 10x + 3 = y$
9. $y + 2 = x^2 - 5x$
10. $(x-3)(x+4) - 7x = y$
11. $-4x^2 + 8x + 3 = y$
12. $0.009x^2 - 0.86x + 2 = y$
13. $y = 2x^3 - 50x$
14. $y = 3x^2 + 4x$

**Answers:**

1. $(3,0), (-5,0)$ and $(0,-15)$
2. $\left(-\frac{1}{2}, 0\right), (-3,0)$ and $(0,3)$
3. $\left(\frac{2}{3}, 0\right), (1,0)$ and $(0,2)$
4. $(0,0), (2,0)$ and $(0,0)$
5. $(7,0), \left(-\frac{5}{2}, 0\right)$, and $(0,-35)$
6. $(5,0), \left(\frac{1}{2}, 0\right)$, and $(0,5)$
7. $\left(-\frac{1}{3}, 0\right), (-2,0)$, and $(0,2)$
8. $\left(-\frac{3}{4}, 0\right), \left(-\frac{1}{2}, 0\right)$, and $(0,3)$
9. $\left(\frac{5 \pm \sqrt{33}}{2}, 0\right)$ or $(\approx 5.37, 0), (\approx -0.37, 0)$, and $(0,-2)$
10. $\left(\frac{-6 \pm \sqrt{84}}{2}, 0\right)$ or $(\approx 7.58, 0), (\approx -1.58, 0)$, and $(0,-12)$
11. $\left(\frac{-8 \pm \sqrt{112}}{-8}, 0\right)$ or $(\approx -0.32, 0), (\approx 2.32, 0)$, and $(0,3)$
12. $\left(\frac{0.86 \pm \sqrt{0.6676}}{0.018}, 0\right)$ or $(\approx 2.39, 0), (\approx 93.17, 0)$, and $(0,2)$
13. $(0,0), (5,0), (-5,0)$, and $(0,0)$
14. $(0,0), \left(-\frac{4}{3}, 0\right)$, and $(0,0)$

# Checkpoint 7B

## Problem 7-131

**Completing the Square to Find the Vertex of a Parabola**

Answers to problem 7-131: Graphing form: $y = 2(x-1)^2 + 3$, vertex $(1, 3)$
See graph in solution that follows examples.

If a quadratic function is in graphing form then the vertex can be found easily and a sketch of the graph can be made quickly. If the equation of the parabola is not in graphing form, the equation can be rewritten in graphing form by completing the square.

First, recall that $y = x^2$ is the parent equation for quadratic functions and the general equation can be written in graphing form as $y = a(x-h)^2 + k$ where $(h, k)$ is the vertex, and relative to the parent graph the function has been:

- Vertically stretched, if the absolute value of *a* is greater than 1
- Vertically compressed, if the absolute value of *a* is less than 1
- Reflected across the *x*-axis, if *a* is less than 0.

**Example 1: Complete the square to change $y = x^2 + 8x + 10$ into graphing form and name the vertex.**

Solution: Use an area model to make $x^2 + 8x$ into a perfect square. To do this, use half of the coefficient of the *x*-term on each side of the area model, and complete the upper right corner of the square, as shown at right.

| | | |
|---|---|---|
| 4 | $4x$ | 16 |
| $x$ | $x^2$ | $4x$ |
| | $x$ | 4 |

Your square shows that $(x+4)^2 = x^2 + 8x + 16$. Your original expression is $x^2 + 8x + 10$, which is 6 fewer than $(x+4)^2$. So you can write $y = x^2 + 8x + 10 = (x+4)^2 - 6$.

Because the function is now in graphing form, $y = a(x-h)^2 + k$, you know the vertex is $(h, k)$, in this case $(-4, -6)$.

**Example 2: Complete the square to change $y = x^2 + 5x + 7$ into graphing form and name the vertex.**

Solution: We need to make $x^2 + 5x$ into a perfect square. Again, we take half of the coefficient of *x* and fill in the upper right of the area model, as shown below.

| | | |
|---|---|---|
| 2.5 | $2.5x$ | 6.25 |
| $x$ | $x^2$ | $2.5x$ |
| | $x$ | 2.5 |

The area model shows that $(x+2.5)^2 = x^2 + 5x + 6.25$.
Your original expression, $y = x^2 + 5x + 7$, is 0.75 more than $(x+2.5)^2$.
So you can write: $y = x^2 + 5x + 7 = (x+2.5)^2 + 0.75$ and the vertex is $(-2.5, 0.75)$.

**Example 3: Complete the square to change $y = 2x^2 - 6x + 2$ into graphing form and name the vertex.**

Solution: This problem is different because the $x^2$ term has a coefficient. First factor the 2 out of the quadratic expression so that an $x^2$ term remains, as follows: $y = 2x^2 - 6x + 2 = 2(x^2 - 3x + 1)$. Then make $x^2 - 3x$ into a perfect square as before:

| | | |
|---|---|---|
| $-1.5$ | $-1.5x$ | $2.25$ |
| $x$ | $x^2$ | $-1.5x$ |
| | $x$ | $-1.5$ |

Since $(x-1.5)^2 = x^2 - 3x + 2.25$, the original expression $x^2 - 3x + 1$ is 1.25 less than $(x-1.5)^2$. You can write:

$y = 2x^2 - 6x + 2$

$y = 2(x^2 - 3x + 1)$

$y = 2\left((x-1.5)^2 - 1.25\right)$

which can be distributed to give

$y = 2(x-1.5)^2 - 2.5$

The vertex is (1.5, –2.5).

Now we can go back and solve the original problem.

Complete the square to change the equation $y = 2x^2 - 4x + 5$ to graphing form, identify the vertex of the parabola, and sketch its graph.

Solution: Factor out the coefficient of $x^2$, resulting in $y = 2\left(x^2 - 2x + \frac{5}{2}\right)$. Make a perfect square out of $x^2 - 2x$:

| | | |
|---|---|---|
| $-1$ | $-1x$ | $1$ |
| $x$ | $x^2$ | $-1x$ |
| | $x$ | $-1$ |

Since $(x-1)^2 = x^2 - 2x + 1$, the original expression $x^2 - 2x + \frac{5}{2}$ is $\frac{3}{2}$ more than $(x-1)^2$. You can write:

$y = 2x^2 - 4x + 5$

$y = 2(x^2 - 2x + \frac{5}{2})$

$y = 2\left((x-1)^2 + \frac{3}{2}\right)$

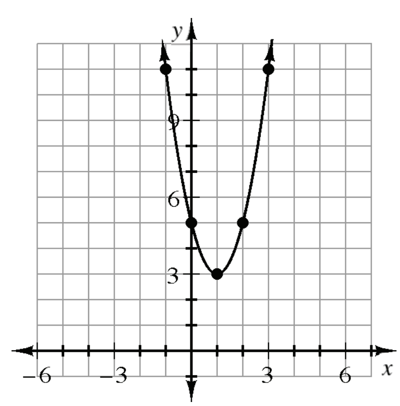
which can be distributed to give

$y = 2(x-1)^2 + 3$

Because the function is now in graphing form, $y = a(x-h)^2 + k$, you know the vertex is $(h, k) = (1, 3)$.

To sketch the graph, we start by plotting the vertex, (1, 3). From the standard form, $y = 2x^2 - 4x + 5$, we see that the $y$-intercept is (0, 5), because when $x = 0$, $y = 5$. By symmetry, (2, 5) must also be a point. Thus we get the graph at the right. If desired, additional points can be found by recognizing that the shape of this parabola is the shape of $y = x^2$ stretched by a factor of 2.

Answer: Graphing form: $y = 2(x-1)^2 + 3$; Vertex: $(1, 3)$; See graph at right.



Here are some more to try. Write each equation in graphing form. If needed, complete the square to do so. Then state the vertex, *y*-intercept, and the stretch factor and sketch a graph.

1. $y = x^2 - 6x + 9$
2. $y = x^2 + 3$
3. $y = x^2 - 4x$
4. $y = x^2 + 2x - 3$
5. $y = x^2 + 5x + 1$
6. $y = x^2 - \frac{1}{3}x$
7. $y = 3x^2 - 6x + 1$
8. $y = 5x^2 + 20x - 16$
9. $y = -x^2 - 6x + 10$

**Answers:**

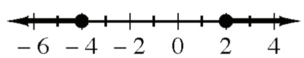
1. $y = (x-3)^2$ ; $(3, 0)$; $(0, 9)$; $a = 1$
2. $y = (x-0)^2 + 3$; $(0, 3)$; $(0, 3)$; $a = 1$
3. $y = (x-2)^2 - 4$ ; $(2, -4)$; $(0, 0)$; $a = 1$
4. $y = (x+1)^2 - 4$ ; $(-1, -4)$; $(0, -3)$ $a = 1$
5. $y = \left(x + \frac{5}{2}\right)^2 - 5\frac{1}{4}$ ; $\left(-\frac{5}{2}, -5\frac{1}{4}\right)$; $(0, 1)$; $a = 1$
6. $y = \left(x - \frac{1}{6}\right)^2 - \frac{1}{36}$ ; $\left(\frac{1}{6}, -\frac{1}{36}\right)$; $(0, 0)$; $a = 1$
7. $y = 3(x-1)^2 - 2$ ; $(1, -2)$ ; $(0, 1)$; $a = 3$
8. $y = 5(x+2)^2 - 36$ ; $(-2, -36)$ ; $(0, -16)$; $a = 5$
9. $y = -(x+3)^2 + 19$ $(-3, 19)$ ; $(0, 10)$; $a = -1$

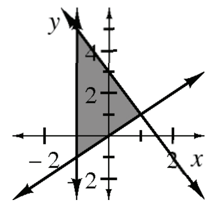
# Checkpoint 8A

## Problem 8-127

**Solving and Graphing Inequalities**

Answers to problem 8-127:

a. 


b. 


There are several methods for graphing different types of inequalities but there is one way that works for all types: Solve as you would an equation then use the solutions to break the graph into regions. If testing a point from a region makes the inequality "true," shade in that region as it is part of the solution. If a point from the region makes the inequality "false," then that region is not part of the solution.

**Example 1: Graph $x^2 + 5x + 4 < 0$ .**

Solution: Change the inequality into an equation and solve.

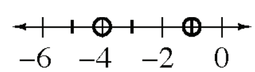
$$x^2 + 5x + 4 = 0$$
$$(x+4)(x+1) = 0$$
$$x = -4 \text{ or } x = -1$$

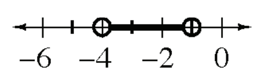
Place the solutions on a number line to break the line into three regions. Use open circles since the original problem is a strict inequality. Test one point from each region in the original inequality. For example testing $x = -5$ we find:

$$(-5)^2 + 5(-5) + 4 < 0$$
$$4 < 0 \text{ False}$$

So that region is not shaded in. Continue in the same manner with one point from each of the other regions.

| test | test | test |
|---|---|---|
| $x = -5$ | $x = -2$ | $x = 0$ |
| F | T | F |

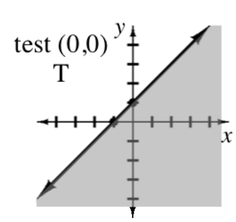


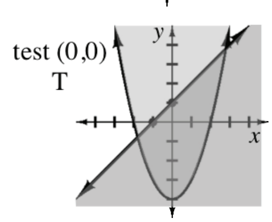

The solution may also be written $-4 < x < -1$ .

**Example 2: Graph the system $y \le x + 1$**
**$y \ge x^2 - 4$**

Solution: Graph the line $y = x + 1$ and check the point $(0, 0)$. The point makes the first inequality "true" so the shading is in the $(0, 0)$ region or below the line.

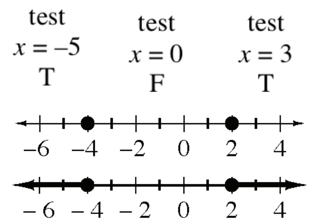



Next graph the parabola $y = x^2 - 4$ and again finding the point $(0, 0)$ to be "true" in the inequality, the shading is inside the parabola. The overlapping shaded region is the solution to the system.



Now we can go back and solve the original problems.

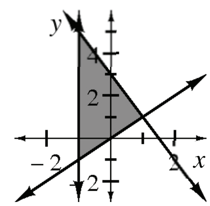
a. $|x+1| \ge 3$




Solution: If $|x+1| = 3$ then $x+1 = \pm 3$ or $x = 2, -4$. Using solid dots to divide the number line into three regions and testing a point from each region gives the answer graph at right. This can also be written as: $x \le -4$ or $x \ge 2$.

b. Graph: $y \le -2x+3$ $\quad y \ge x$ $\quad x \ge -1$

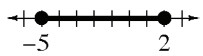
Solution: Start by looking at the equation of the line that marks the edge of each inequality. The first has slope $-2$ and $y$-intercept $(0, 3)$. Checking $(0, 1)$ gives a true statement so we shade below the solid line. The second line has slope 1 and $y$-intercept $(0, 0)$. Again checking $(0, 1)$ gives a true statement, so we shade above the solid line. The third is a vertical line at $x = -1$. Checking a point tells us to shade the right side. The overlapping shading is a triangle with vertices $(-1, 5)$, $(1, 1)$, and $(-1, -1)$. See the answer graph at right.

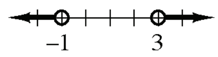


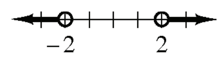

Here are a few more to try. In problems 1 through 6, graph each inequality.
In problems 7 through14 graph the solution region for each system of inequalities.

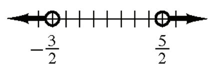
1. $|2x+3| \le 7$
2. $x^2 - 2x - 3 > 0$
3. $4 - x^2 \le 0$
4. $|4r-2| > 8$
5. $3m^2 \le 9m$
6. $-|x+3| < 10$
7. $y \le -x+2$, $y \le 3x-6$
8. $y \ge \frac{2}{3}x+4$, $y \le \frac{7}{12}x+5$
9. $x < 3$, $y \ge -2$
10. $y \le 4x+16$, $y \ge -\frac{4}{3}x-4$
11. $y \ge x^2-4$, $y < -3x+1$
12. $y < 2x+5$, $y \ge |x+1|$
13. $y < x^2-2x-3$, $y \le \frac{1}{2}x+3$
14. $y \le -\frac{1}{2}x+2$, $y \ge (x+1)^2-3$

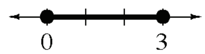
**Answers:**

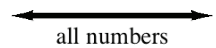
1. 

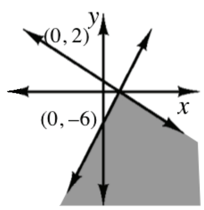

2. 

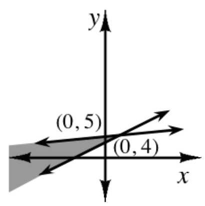

3. 

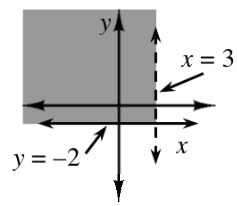

4. 

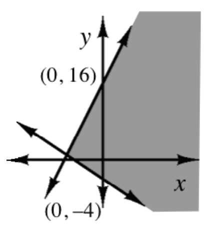

5. 

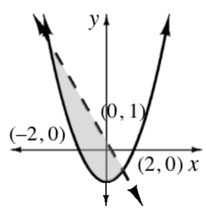

6. 

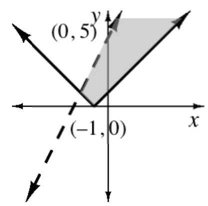

7. 

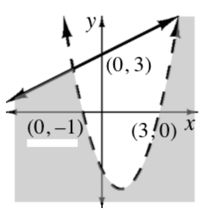

8. 

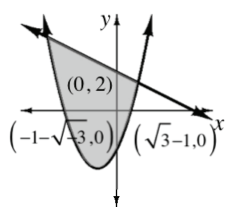

9. 


10. 


11. 


12. 


13. 


14. 

# Checkpoint 8B

## Problem 8-174

### Solving Complicated Equations

Answers to problem 8-174: a: $x = 5$ or $1$, b: $x = 4$ or $0$, c: $x = 7$, d: $x = 1$

Often the best way to solve a complicated equation is to use a method such as the Looking Inside or Undoing Methods. Checking your answer(s) is important because sometimes solutions are accurately found but will not work in the original equation. These answers are called extraneous solutions.

**Example 1: Solve $3(x+1)^3 - 1 = 80$**

Solution: In this case we will undo everything.

| | | check |
|---|---|---|
| $3(x+1)^3 - 1 = 80$ | original problem | $3(2+1)^3 = 81$ |
| $3(x+1)^3 = 81$ | add 1 to undo $-1$ | $3(3)^3 = 81$ |
| $(x+1)^3 = 27$ | division undoes multiplication | $3(27) = 81$ |
| $x+1 = 3$ | cube root undoes cubing | $81 = 81$ |
| $x = 2$ | subtract 1 to undo $+1$ | |

**Example 2: Solve $|2x+5| = x+4$**

Solution: In this case we will use the Look Inside Method and the fact that absolute value of a quantity and its opposite are the same.

$$2x+5 = x+4 \quad \text{or} \quad -(2x+5) = x+4$$
$$x = -1 \qquad\qquad -2x-5 = x+4$$
$$-3x = 9$$
$$x = -3$$

checks

$$|2(-1)+5| = -1+4 \qquad |2(-3)+5| = -3+4$$
$$|3| = |3| \qquad |-1| = |1|$$
$$3 = 3 \text{ True} \qquad 1=1 \text{ True}$$

The solutions are $x = -1, x = -3$.

Now we can go back and solve the original problems.

a.
$$2|x-3|+7=11$$
$$2|x-3|=4$$
$$|x-3|=2$$
$$x-3=2 \text{ or } -(x-3)=2$$
$$x=5 \text{ or } x=1$$

checks

$$2|5-3|+7=11 \qquad 2|1-3|+7=11$$
$$2|2|+7=11 \qquad 2|-2|+7=11$$
$$4+7=11 \text{ True} \qquad 4+7=11 \text{ True}$$

The solutions are $x=5$, $x=1$.

b.
$$4(x-2)^2=16$$
$$(x-2)^2=4$$
$$x-2=\pm 2$$
$$x=2\pm 2=4 \text{ or } 0$$

checks

$$4(4-2)^2=16 \qquad 4(0-2)^2=16$$
$$4(2)^2=16 \qquad 4(-2)^2=16$$
$$4(4)=16 \text{ True} \qquad 4(4)=16 \text{ True}$$

The solutions are $x=4$, $x=0$.

c.
$$\sqrt{x+18}=x-2$$
$$(\sqrt{x+18})^2=(x-2)^2$$
$$x+18=x^2-4x+4$$
$$0=x^2-5x-14$$
$$0=(x-7)(x+2)$$
$$x=7 \text{ or } x=-2$$

checks

$$\sqrt{7+18}=7-2 \qquad \sqrt{-2+18}=-2-2$$
$$\sqrt{25}=5 \text{ True} \qquad \sqrt{16}=-4 \text{ False}$$

The only solution is $x=7$.

d.
$$|2x+5|=3x+4$$
$$2x+5=3x+4 \text{ or } -(2x+5)=3x+4$$
$$x=1 \qquad \text{or} \qquad x=-\tfrac{9}{5}$$

checks

$$|2(1)+5|=3(1)+4 \qquad |2(-\tfrac{9}{5})+5|=3(-\tfrac{9}{5})+4$$
$$|7|=7 \text{ True} \qquad |-\tfrac{18}{5}+\tfrac{25}{5}|=-\tfrac{27}{5}+\tfrac{20}{5}$$
$$|\tfrac{7}{5}|=-\tfrac{7}{5} \text{ False}$$

The only solution is $x=1$.

Here are some more to try. Solve each equation and check your solutions.

1. $|x+4|+3=17$
2. $|3w-6|-2=19$
3. $\sqrt{3w+4}-2=2$
4. $\sqrt{3x+13}=x+5$
5. $\sqrt[3]{x}-2=5$
6. $(x+1)^2+1=6$
7. $|2y-4|=12$
8. $|3m+5|=5m+2$
9. $\sqrt{y+7}+5=y$
10. $\sqrt{5-m}=m+1$
11. $\frac{2(x-1)^2}{3}-7=1$
12. $3(x-2)^3=81$
13. $|2m+5|=m+4$
14. $|2y+8|=3y+7$
15. $\sqrt{x+7}-x=1$
16. $\sqrt{y+2}=y$
17. $\sqrt[4]{x+1}+2=5$
18. $\frac{1}{2}(x+5)^3+1=10$
19. $\sqrt{x}+2=x$
20. $\sqrt{x}+2=\sqrt{x+6}$

**Answers:**

1. $x=-18, 10$
2. $w=-5, 9$
3. $w=4$
4. $x=-3, -4$
5. $x=343$
6. $x=-1\pm\sqrt{5}$
7. $y=-4, 8$
8. $m=\frac{3}{2}$
9. $y=9$
10. $m=1$
11. $x=1\pm\sqrt{12}$
12. $x=5$
13. $m=-1, -3$
14. $y=1$
15. $x=2$
16. $y=2$
17. $x=80$
18. $x=-5+\sqrt[3]{18}$
19. $x=4$
20. $x=\frac{1}{4}$

# Checkpoint 9A

## Problem 9-41

### Writing and Solving Exponential Equations

Answers to problem 9-41:

a. The more rabbits you have, the more new ones you get, a linear model would grow by the same number each year. A sine function would be better if the population rises and falls, but more data would be needed to apply this model.

b. $R = 80,000(5.4772...)^t$ c. $\approx 394$ million

d. 1859, it seems okay that they grew to 80,000 in 7 years, *if* they are growing exponentially.

e. No, since it would predict a huge number of rabbits now. The population probably leveled off at some point or dropped drastically and rebuilt periodically.

Exponential functions are equations of the form $y = ab^x + c$ where $a$ represents the initial value, $b$ represents the multiplier, $c$ represents the horizontal asymptote, and $x$ often represents the time. Some problems simply involve substituting in the given information into the equation and then doing calculations. If you are trying to solve for the time ($x$), then you will usually need to use logarithms. If you need to find the multiplier ($m$), then you will need to find a root. Note that we often assume that $c = 0$, unless we are told otherwise.

**Example 1: Lunch at our favorite fast food stands cost \$6.50. The price has steadily increased 4% per year for many years.**

**Question 1: What will lunch cost in 10 years?**

Solution: In this case, we can use \$6.50 as the initial value and the multiplier is 1.04, so the equation for the situation is $y = 6.50(1.04)^x$. The time we are interested in here is 10 years. Substituting into the equation, $y = 6.50(1.04)^{10} = \$9.62$.

**Question 2: What did it cost 10 years ago?**

Solution: Using the same equation, only using –10 for the years, $y = 6.50(1.04)^{-10} = \$4.39$.

**Question 3: How long before lunch costs \$10?**

Solution: Again, we use the same equation, but this time we know the $y$-value, but not the value of $x$. To solve for $x$, we use logarithms, as shown in the work at right.

Answer: About 11 years.

$$\$10 = 6.50(1.04)^x$$
$$1.04^x = \tfrac{10}{6.5}$$
$$\log 1.04^x = \log\left(\tfrac{10}{6.5}\right)$$
$$x \log 1.04 = \log\left(\tfrac{10}{6.5}\right)$$
$$x = \frac{\log\left(\frac{10}{6.5}\right)}{\log 1.04} \approx 11$$

**Example 2: Tickets for a big concert first went on sale three weeks ago for \$60. This week people are charging \$100. Write an equation that represents the cost of the tickets *w* weeks from the time that they went on sale. Assume that they continue to increase in the same way.**

Solution: To find the multiplier, we can use what we are given. The initial value is \$60, the time is 3 weeks, and the final value is \$100. This gives $100 = 60k^3$. Solving for $k$ gives $k = \sqrt[3]{\frac{100}{60}} \approx 1.186$.

Answer: The equation is approximately $y = 60(1.186)^w$.

Now we can go back and solve parts (b), (c), and (d) of the original problem.

When rabbits were first brought to Australia, they had no natural enemies. There were about 80,000 rabbits in 1866. Two years later, in 1868, the population had grown to over 2,400,000!

b. Write an exponential equation for the number of rabbits $t$ years after 1866.

Solution: For 1866, 80000 would be the initial value, time would be 2 years, and the final amount would be 2400000, which gives the equation $2400000 = 80000m^2$. Solving for $m$, we get $30 = m^2$ so $m = \sqrt{30} \approx 5.477$. Thus the equation is approximately $R = 80{,}000(5.477)^t$.

c. How many rabbits do you predict would have been present in 1871?

Solution: The initial value is still 80,000, the multiplier $\approx 5.477$, and now the time is 5 years. This gives $80{,}000(5.477)^5 \approx$ 394 million.

d. According to your model, in what year was the first pair of rabbits introduced into Australia?

Solution: In this situation, we use 2 as the initial value, 80000 as the final value, and the multiplier is still 5.477 but now the time is not known. Using these values, we get $80000 = 2(5.477)^x$, which is solved at right. The answer 6.23 tells us that approximately 6.23 years had passed between the time of the first pair of rabbits, and the time when there were 80000. Thus, rabbits would have been introduced sometime in 1859.

$$80000 = 2(5.477)^x$$
$$40000 = (5.477)^x$$
$$\log(5.477)^x = \log 40000$$
$$x \log(5.477) = \log 40000$$
$$x = \frac{\log(40000)}{\log(5.477)} \approx 6.23$$

Here are some more to try:

1. A DVD loses 60% of its value every year it is in the store. The DVD costs $80 new. Write a function that represents its value in *t* years. What is it worth after 4 years?

2. Inflation is at a rate of 7% per year. Evan's favorite bread now costs $1.79. What did it cost 10 years ago? How long before the cost of the bread doubles?

3. A bond that appreciates 4% per year will be worth $146 in five years. Find the current value.

4. Sixty years ago, when Sam's grandfather was a kid, he could buy his friend dinner for $1.50. If that same dinner now costs $25.25 and inflation was consistent, write an equation that models the cost for the dinner at different times.

5. A two-bedroom house in Omaha is now worth $110,000. If it appreciates at a rate of 2.5% per year, how long will it take to be worth $200,000?

6. A car valued at $14,000 depreciates 18% per year. After how many years will the value have depreciated to $1000?

**Answers:**

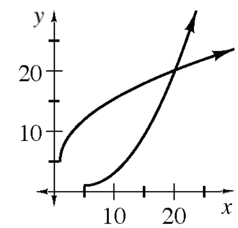
1. $y = 80(0.4)^t$ , $2.05
2. $0.91, 10.2 years
3. $120
4. $y = 1.50(1.048)^x$
5. 24.2 years
6. 13.3 years

# Checkpoint 9B

## Problem 9-111

### Finding the Equation for the Inverse of a Function

Answers to problem 9-111:




a. $f^{-1}(x) = \frac{1}{3}\left(\frac{x-5}{2}\right)^2 + 1 = \frac{1}{12}(x-5)^2 + 1$ for $x \geq 5$

b. See graph at right.

To find the equation for the inverse of a function, you can interchange the $x$ and $y$ variables and then solve for $y$. This also means that the coordinates of points that are on the graph of the function will be reversed on the graph of the inverse. Here are some examples:

**Example 1: Write the equation for the inverse of $y = 2(x+3)$.**

Solution: We can interchange the $x$ and the $y$ to get the equation of the inverse. To get our final answer, we solve for $y$, as shown at right.

$$2(y+3) = x$$
$$(y+3) = \frac{x}{2}$$
$$y = \frac{x}{2} - 3$$

Answer: $y = \frac{x}{2} - 3$

**Example 2: Write the equation for the inverse of $y = \frac{1}{2}(x+4)^2 + 1$.**

Solution: Again, we can interchange the $x$ and the $y$ to get the equation of the inverse and then solve for $y$ to get our answer in $y$= form, as shown at right.

$$x = \frac{1}{2}(y+4)^2 + 1$$
$$\frac{1}{2}(y+4)^2 = x - 1$$
$$(y+4)^2 = 2x - 2$$
$$y + 4 = \pm\sqrt{2x-2}$$
$$y = -4 \pm \sqrt{2x-2}$$

Answer: $y = -4 \pm \sqrt{2x-2}$. Note that because of the $\pm$, this inverse is not a function.

**Example 3: Write the equation for the inverse of $y = -\frac{2}{3}x + 6$.**

Solution: Interchanging the $x$ and the $y$, we get $x = -\frac{2}{3}y + 6$. Solving for $y$ gives $y = -\frac{3}{2}(x-6) = -\frac{3}{2}x + 9$.

Answer: $y = -\frac{3}{2}x + 9$

**Example 4: Write the equation for the inverse of $y = \sqrt{x-2} + 3$.**

Solution: Again, we exchange $x$ and $y$ and then solve for $y$, as shown at right.

$$x = \sqrt{y-2} + 3$$
$$\sqrt{y-2} = x - 3$$
$$y - 2 = (x-3)^2$$
$$y = (x-3)^2 + 2$$

The original function is half of a sleeping parabola, so the inverse is only half of a parabola as well. Thus the domain of the inverse is restricted to $x \geq 3$.

Answer: $y = (x-3)^2 + 2$ in the domain $x \geq 3$.

Now we can go back and solve the original problem.

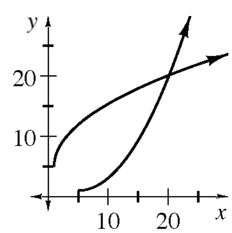
Find the equation for the inverse of the function $y = 2\sqrt{3(x-1)} + 5$. Then sketch the graph of both the original and the inverse.

Solution: Interchanging $x$ and $y$ we get $x = 2\sqrt{3(y-1)} + 5$. We then solve for $y$, as shown at right. This equation is simplified to get $y = \frac{(x-5)^2}{12} + 1$.

$$x = 2\sqrt{3(y-1)} + 5$$
$$2\sqrt{3(y-1)} = x - 5$$
$$\sqrt{3(y-1)} = \tfrac{x-5}{2}$$
$$3(y-1) = \left(\tfrac{x-5}{2}\right)^2$$
$$y - 1 = \tfrac{1}{3}\left(\tfrac{x-5}{2}\right)^2$$
$$y = \tfrac{1}{3}\left(\tfrac{x-5}{2}\right)^2 + 1$$

Note that the domain and range of the inverse are the interchanged domain and range of the original function. In other words, the original function has a domain of $x \geq 1$ and range of $y \geq 5$. The domain of the inverse, then, is $x \geq 5$ and the range is $y \geq 1$.

As you can see by the graph at right, the points on the inverse graph, have interchanged coordinates from the points on the graph of the original function. For example, two points on the original graph are (1, 5) and (4, 11). The corresponding points on the graph of the inverse are (5, 1) and (11, 4).




Answer: $y = \frac{(x-5)^2}{12} + 1$ in the domain $x \geq 5$.

Here are some more to try. Find an equation for the inverse of each function.

1. $y = 3x - 2$
2. $y = \frac{x+1}{4}$
3. $y = \frac{1}{3}x + 2$
4. $y = x^3 + 1$
5. $y = 1 + \sqrt{x+5}$
6. $y = 3(x+2)^2 - 7$
7. $y = 2\sqrt{x-1} + 3$
8. $y = \frac{1}{2+x}$
9. $y = \log_3(x+2)$

**Answers:**

1. $y = \frac{x+2}{3}$
2. $y = 4x - 1$
3. $y = 3x - 6$
4. $y = \sqrt[3]{x-1}$
5. $y = (x-1)^2 - 5$
6. $y = -2 \pm \sqrt{\frac{x+7}{3}}$
7. $y = \left(\frac{x-3}{2}\right)^2 + 1$
8. $y = \frac{1}{x} - 2$
9. $y = 3^x - 2$

# Checkpoint 10

## Problem 10-176

**Rewriting Expressions with and Solving Equations with Logarithms**

Answers to problem 10-176: a: $\log_2(5x)$, b: $\log_2(5x^2)$, c: 17, d: $-\frac{9}{20} = 0.45$, e: 15, f: 4

First a review of the properties based on the inverse relationship of exponentials and logarithms:

The following definitions and properties hold true for all positive $m \neq 1$.

| | |
|---|---|
| Definition of logarithms: | $\log_m(a) = n$ means $m^n = a$ |
| Product Property: | $\log_m(a \cdot b) = \log_m(a) + \log_m(b)$ |
| Quotient Property: | $\log_m(\frac{a}{b}) = \log_m(a) - \log_m(b)$ |
| Power Property: | $\log_m(a^n) = n \cdot \log_m(a)$ |
| Inverse relationship: | $\log_m(m)^n = n$ and $m^{\log_m(n)} = n$ |

**Example 1: Write as a single logarithm: $3\log_5(x) + \log_5(x+1)$**

Solution:

$$
\begin{aligned}
3\log_5(x) + \log_5(x+1) &= \log_5(x^3) + \log_5(x+1) && \text{by the Power Property} \\
&= \log_5(x^3(x+1)) && \text{by the Product Property} \\
&= \log_5(x^4 + x^3) && \text{by simplifying}
\end{aligned}
$$

**Example 2: Solve $\log_2(x) - \log_2(3) = 4$**

Solution:

$$
\begin{aligned}
\log_2(x) - \log_2(3) &= 4 && \text{problem} \\
\log_2\left(\tfrac{x}{3}\right) &= 4 && \text{by the Quotient Property} \\
2^4 &= \tfrac{x}{3} && \text{by the definition of logarithms} \\
48 &= x && \text{multiply both sides by 3}
\end{aligned}
$$

Now we can go back and solve the original problems.

a. $\log_2(30x) - \log_2(6)$

$\log_2(\frac{30x}{6}) = \log_2(5x)$

b. $2\log_3(x) + \log_3(5)$

$\log_3(x^2) + \log_3(5)$

$\log_3(x^2 \cdot 5) = \log_3(5x^2)$

c. $\log_7(3x-2) = 2$

$7^2 = 3x - 2$

$49 = 3x - 2 \Rightarrow x = 17$

d. $\log(2x+1) = -1$

$10^{-1} = 2x + 1$

$\frac{1}{10} = 2x + 1 \Rightarrow x = -\frac{9}{20} = 0.45$

e. $\log_5(3y) + \log_5(9) = \log_5(405)$

$\log_5(3y \cdot 9) = \log_5(405)$

$\log_5(27y) = \log_5(405)$

$27y = 405 \Rightarrow y = 15$

f. $\log(x) + \log(x+21) = 2$

$\log(x^2 + 21x) = 2$

$x^2 + 21x = 10^2$

$x^2 + 21x - 100 = 0$

$(x+25)(x-4) = 0$

$x = -25,\ x = 4$

–25 is an extraneous solution so $x = 4$ only

Here are some more to try. In problems 1 through 8, write each expression as a single logarithm. In problems 9 through 26 solve each equation.

1. $\log_3(5)+\log_3(m)$
2. $\log_2(q)-\log_2(z)$
3. $\log_6(r)+3\log_6(x)$
4. $\log(90)+\log(4)-\log(36)$
5. $\log_4(16x)-\log_4(x)$
6. $\log(\sqrt{x})+\log(x^2)$
7. $\log_5(\sqrt{x})+\log_5(\sqrt[3]{x})$
8. $\log_5(x-1)+\log_5(x+1)$
9. $\log_4(2x+3)=\frac{1}{2}$
10. $\log_5(3x+1)=2$
11. $\log_9(9^2)=x$
12. $16^{\log_{16}(5)}=y$
13. $8^{\log_8(x)}=3$
14. $\log_5(5^{0.3})=y$
15. $\log_2(x)=3\log_2(4)+\log_2(5)$
16. $\log_6(x)+\log_6(8)=\log_6(48)$
17. $\log_2(144)-\log_2(x)=\log_2(9)$
18. $\log_2(36)-\log_2(y)=\log_2(12)$
19. $\log_5(3x-1)=-1$
20. $\log_2(x)-\log_2(3)=4$
21. $\frac{1}{3}\log(3x+1)=2$
22. $\log_2(5)+\frac{1}{2}\log_2(x)=\log_2(15)$
23. $\frac{1}{2}\log(y)=2\log(2)+\log(16)$
24. $\log_2(x^2+2x)=3$
25. $2\log_4(x)-\log_4(3)=2$
26. $\log_7(x+1)+\log_7(x-5)=1$

**Answers:**

1. $\log_3(5m)$
2. $\log_2(\frac{q}{z})$
3. $\log_6(rx^3)$
4. $\log(10)=1$
5. $\log_4(16)=2$
6. $\log(x^{5/2})$
7. $\log_5(x^{5/6})$
8. $\log_5(x^2-1)$
9. $-\frac{1}{2}$
10. 8
11. 2
12. 5
13. 3
14. 0.3
15. 320
16. 6
17. 16
18. 3
19. $\frac{2}{5}$
20. 48
21. 333,333
22. 9
23. 4096
24. –4, 2
25. $\sqrt{48}=4\sqrt{3}$
26. 6

# Checkpoint 11

## Problem 11-95

### Solving Rational Equations

Answers to problem 11-95: a: $x = \pm 2\sqrt{3}$ , b: $x = 2$ , c: $x = \frac{2}{9}$, d: $x = \frac{-1 \pm \sqrt{13}}{6} \approx 0.434$ or $-0.768$

To solve rational equations (equations with fractions) it is usually best to first multiply everything by the common denominator to remove the fractions, a method known as **Fraction Busters**. After you have done this, you can solve the equation using your usual strategies. Following are a few examples.

**Example 1: Solve $\frac{24}{x+1} = \frac{16}{1}$.**

Solution: The common denominator in this case is $(x+1)$. Multiplying both sides of the equation by $(x+1)$ removes all fractions from the equation. You can then simplify and solve for $x$. This process is demonstrated at right.

$$(x+1)\left(\frac{24}{x+1}\right) = (x+1)\left(\frac{16}{1}\right)$$
$$24 = 16(x+1)$$
$$24 = 16x + 16$$
$$8 = 16x$$
$$x = \frac{1}{2}$$

Answer: $x = \frac{1}{2}$

**Example 2: Solve $\frac{5}{2x} + \frac{1}{6} = 8$.**

Solution: Again, we multiply both sides of the equation by the common denominator, which, in this case, is $6x$. We must be careful to remember to distribute so that we multiply each term on both sides of the equation by $6x$. Then we simplify and solve, as shown at right.

$$6x\left(\frac{5}{2x} + \frac{1}{6}\right) = 6x(8)$$
$$6x \cdot \frac{5}{2x} + 6x \cdot \frac{1}{6} = 48x$$
$$15 + x = 48x$$
$$15 = 47x$$
$$x = \frac{15}{47}$$

Answer: $x = \frac{15}{47}$

Now we can go back and solve the original problems.

a:

$\frac{x}{3} = \frac{4}{x}$

$3x\left(\frac{x}{3}\right) = 3x\left(\frac{4}{x}\right)$

$x^2 = 12$

$x = \pm\sqrt{12} = \pm 2\sqrt{3}$

$x \approx \pm 3.46$

b:

$\frac{x}{x-1} = \frac{4}{x}$

$x(x-1)\left(\frac{x}{x-1}\right) = x(x-1)\left(\frac{4}{x}\right)$

$x^2 = 4(x-1)$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2$

c:

$\frac{1}{x} + \frac{1}{3x} = 6$

$3x\left(\frac{1}{x} + \frac{1}{3x}\right) = 3x(6)$

$3x\left(\frac{1}{x}\right) + 3x\left(\frac{1}{3x}\right) = 18x$

$3 + 1 = 18x$

$x = \frac{2}{9}$

d:

$\frac{1}{x} + \frac{1}{x+1} = 3$

$x(x+1)\left(\frac{1}{x} + \frac{1}{x+1}\right) = x(x+1)(3)$

$x(x+1)\left(\frac{1}{x}\right) + x(x+1)\left(\frac{1}{x+1}\right) = x(x+1)(3)$

$x + 1 + x = 3x^2 + 3x$

$0 = 3x^2 + x - 1$

Using the Quadratic Formula,
$x \approx -0.434, -0.768$

Here are some more to try. Solve each of the following rational equations.

1. $\frac{3x}{5} = \frac{x-2}{4}$
2. $\frac{4x-1}{x} = 3x$
3. $\frac{2x}{5} - \frac{1}{3} = \frac{137}{3}$
4. $\frac{4x-1}{x+1} = x - 1$
5. $\frac{x}{3} = x + 4$
6. $\frac{x-1}{5} = \frac{3}{x+1}$
7. $\frac{x+6}{3} = x$
8. $\frac{2x+3}{6} + \frac{1}{2} = \frac{x}{2}$
9. $\frac{3}{x} + \frac{5}{x-7} = -2$
10. $\frac{2x+3}{4} - \frac{x-7}{6} = \frac{2x-3}{12}$

**Answers:**

1. $x = -\frac{10}{7}$
2. $x = \frac{1}{3}, 1$
3. $x = 115$
4. $x = 0, 4$
5. $x = -6$
6. $x = \pm 4$
7. $x = 3$
8. $x = 6$
9. $x = \frac{3 \pm \sqrt{51}}{2}$
10. $x = -13$