

Chapter 6 Assessment. Complete these problems to make sure that you are prepared for the test.

1. If $f(x) = 2x^3 + 7$ and $g(x) = -3(x - 9)$, find the value of each expression below.

$$\begin{aligned} a.) f(-3) &= 2(-3)^3 + 7 \\ &= 2(-27) + 7 \quad \text{permas} \\ &= -54 + 7 \\ &= \boxed{-47} \end{aligned}$$

this is a composition

$$\begin{aligned} b.) g(-3) &= -3(-3 - 9) \\ &= -3(-12) \\ &= \boxed{36} \end{aligned}$$

you can dist... but you can also just simplify

$$c.) f(g(-3)) \quad \text{notic } g(-3) = 36$$

$$\begin{aligned} f(36) &= 2(36)^3 + 7 \\ &= 2(46,656) + 7 \\ &= \boxed{93,319} \end{aligned}$$

you need a calculator

$$\begin{aligned} d.) g(f(-3)) &= -3(-47) - 9 \\ &= -3(-56) \\ &= \boxed{168} \end{aligned}$$

- e.) Are $f(x)$ and $g(x)$ inverses of each other? Justify your answer.

No, if they were inverse $f(g(-3))$ would equal -3 ... but it does not!

2. Simplify the following expressions:

$$a.) \left(\frac{4}{x^2}\right)^3 \cdot \frac{x^2y}{8x^3} \cdot \frac{x^2y^2}{4x^2y}$$

$$\frac{4}{x^2} \cdot \frac{4}{x^2} \cdot \frac{4}{x^2} \cdot \frac{x^2y}{8x^3} \cdot \frac{x^2y^2}{4x^2y}$$

$$\begin{aligned} &\frac{64x^4y^3}{32x^7y^4} \cdot \frac{2x^{-7}y^2}{x^{-1}} \\ &\boxed{\frac{2y^2}{x^7}} \end{aligned}$$

$$b.) \frac{x^2 - 4x + 3}{x^2 - 9} \div \frac{6x^2 - x - 2}{x^2 - 4x - 21}$$

$$\frac{x^2 - 4x + 3}{x^2 - 9} \cdot \frac{x^2 - 4x - 21}{6x^2 - x - 2}$$

$$\frac{(x-3)(x-1)}{(x-3)(x+1)} \cdot \frac{(x-7)(x+3)}{(2x+1)(3x-2)}$$

$$\boxed{\frac{(x-1)(x-7)}{(2x+1)(3x-2)}}$$

KCF

Factor

Make big $\boxed{1^{13}}$

$$\begin{aligned} 6x^2 - x - 2 &\quad 12x \\ 6x^2 - 4x - 3x - 2 &\quad \cancel{4x} \cancel{-x} \\ (6x^2 - 4x)(3x - 2) & \\ 2x(3x-2) + 1(3x-2) & \\ (2x+1)(3x-2) & \end{aligned}$$

3. Subtract and simplify the following expressions:

a.) $\frac{x-2}{x+5} - \frac{x-4}{x-3}$

$$\frac{(x-3)(x-2)}{(x-3)(x+5)} - \frac{(x-4)(x+5)}{(x-3)(x+5)}$$

$$\frac{x^2 - 5x + 6}{x^2 + 2x - 15} - \frac{x^2 + x - 20}{x^2 + 2x - 15}$$

$$\begin{array}{r} -6x + 26 \\ \hline x^2 + 2x - 15 \end{array}$$

b.) $\frac{1}{2} + \frac{x-1}{x^2 + 2x - 3} - \frac{x+1}{2x+6}$

$$\frac{1}{2} + \frac{x-1}{(x+3)(x-1)} - \frac{x+1}{2(x+3)}$$

$$\frac{(x+3)}{(x+3)} \cdot \frac{1}{2} + \frac{1}{x+3} \cdot \frac{2}{2} - \frac{x+1}{2(x+3)}$$

$$\frac{x+3}{2(x+3)} + \frac{2}{2(x+3)} - \frac{x+1}{2(x+3)}$$

$$\frac{x+3+2-x-1}{2(x+3)} = \frac{4}{2(x+3)} = \boxed{\frac{2}{x+3}}$$

4. Identify the vertex and the line of symmetry: $f(x) = 5x^2 + 20x - 16$

$$y = a(x-h)^2 + k$$

$$f(x) = 5x^2 + 20x - 16$$

$$\text{complete the square: } (x+2)^2$$

$$x^2 + 4x + 4$$

$$f(x) = 5(x^2 + 4x + 4) - 16 = 20$$

notice the
5 is multiplying
(x+2)

$$f(x) = 5(x+2)^2 - 36$$

$$\text{check: } 5(x^2 + 4x + 4) - 36$$

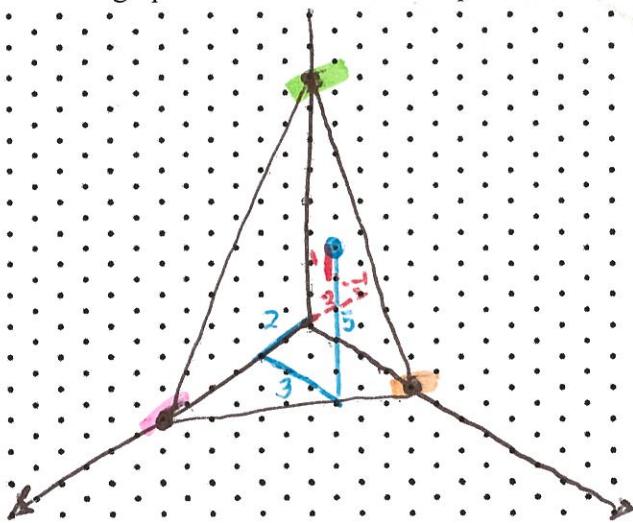
$$5x^2 + 20x + 20 - 36$$

$$5x^2 + 20x - 16$$

$$\rightarrow \text{vertex: } (-2, -36)$$

$$\text{line of symmetry } (x = -2)$$

5. Graph on Isometric graph paper

a. Sketch a graph of the solution to the equation: $4x + 6y + 3z = 24$ 

$$\begin{aligned} &\text{Find } x\text{-int, } y\text{-int, } z\text{-int} \\ &x\text{-int: } y=0, z=0 \\ &4x = 24 \\ &(6, 0, 0) \end{aligned}$$

$$y\text{-int: } x=0, z=0$$

$$6y = 24$$

$$(0, 4, 0)$$

$$z\text{-int: } x=0, y=0$$

$$3z = 24$$

$$(0, 0, 8)$$

b. Graph the point (2,3,5). What are three other points that have the same isometric dot position?

$$(4, 5, 7)$$

$$(3, 4, 6)$$

$$(2, 3, 5)$$

$$(1, 2, 4)$$

$$(0, 1, 3)$$

$$(-1, 0, 2)$$

$$(-2, -1, 1)$$

Same isometric
different 3-D locations

6. Solve for x, y , and z in the system of equations at below.

$$\begin{array}{l} \textcircled{1} \quad x - 2y + 3z = 3 \\ \textcircled{2} \quad 2x + y + 5z = 8 \\ \textcircled{3} \quad 3x - y - 3z = -22 \end{array}$$

label to keep track

Add equations to cancel 1 variable
(you can pick)

$$\begin{array}{r} \textcircled{2} \quad 2x + y + 5z = 8 \\ + \textcircled{3} \quad 3x - y - 3z = -22 \\ \hline \textcircled{4} \quad 5x + 2z = -14 \end{array}$$

I cancelled "y" Notice: multiply $\textcircled{3}$ by 2 so "y" cancels

Name:
Period:

$$\begin{array}{r} \textcircled{1} \quad x - 2y + 3z = 3 \\ 2\textcircled{2} \quad 4x + 2y + 10z = 16 \\ \hline \textcircled{5} \quad 5x + 13z = 19 \end{array}$$

Step 1

Notice: we need two equations!

$$\begin{array}{r} \textcircled{A} \quad 5x + 2z = -14 \\ - \textcircled{B} \quad -5x - 13z = 19 \\ \hline \uparrow \quad -11z = -33 \end{array}$$

negative of \textcircled{B}
why?

... to cancel the x

$$5x + 2(3) = -14$$

$$5x = -20$$

$$x = -4$$

Could we have plugged
into $5x + 13z = 19$?

Yes, it does not matter

$$x - 2y + 3z = 3$$

$$(-4) - 2y + 3(3) = 3$$

$$-2y + 5 = 3$$

$$-2y = -2$$

$$y = 1$$

$$\boxed{(-4, 1, 3)}$$

7. Solve each equation below for x .

a. $18 = 2(3)^{(x+3)}$

$$9 = 3^{x+3}$$

"divide by 2"

$$\log 9 = \log 3^{x+3}$$

"log both sides"

$$\log 9 = (x+3) \log 3$$

"power property"

$$\frac{\log 9}{\log 3} = x+3$$

"divide log 3"

$$-3 + \frac{\log 9}{\log 3} = x$$

"subtract 3"

$$\text{calc } \rightarrow x = -1$$

b. $128 = 4^{3x}$

$$2^7 = (2^2)^{3x}$$

$$2^7 = 2^{6x}$$

$$7 = 6x \quad \text{drop the base}$$

$$\frac{7}{6} = x$$

"common base"
method

$$128 = 4^{3x}$$

$$\log 128 = \log 4^{3x}$$

$$\log 128 = 3x \cdot \log 4$$

$$\frac{\log 128}{\log 4} = 3x$$

$$\frac{\log 128}{\log 4} = x$$

"log method"

$$x = 1.166$$

($7 = 1.166$ as well)

8. What is the equation of the parabola passing through the points $(3, 1), (2, 3)$, and $(0, -5)$?

Plug into $y = ax^2 + bx + c$

$$1 = a(3)^2 + b(3) + c \rightarrow 1 = 9a + 3b + c$$

$$3 = a(2)^2 + b(2) + c \rightarrow 3 = 4a + 2b + c$$

$$-5 = a(0)^2 + b(0) + c \rightarrow c = -5$$

$$1 = 9a + 3b - 5 \rightarrow 6 = 9a + 3b \xrightarrow{+2} 12 = 18a + 6b$$

$$3 = 4a + 2b - 5 \rightarrow 8 = 4a + 2b \xrightarrow{-3} -24 = -12a - 6b$$

$$-12 = 6a$$

$$-2 = a$$

$$1 = 9a + 3b + c$$

$$1 = 9(-2) + 3b - 5$$

$$1 = -18 + 3b - 5$$

$$1 = -23 + 3b$$

$$24 = 3b$$

$$8 = b$$

$$\boxed{(-2, 8, -5)}$$

9. In parts (a) through (b), rewrite each expression as a single logarithm. In parts (c) through (f), solve each equation.

a. $\log_2(30x) - \log_2(6)$ $\xrightarrow{\text{quotient}} \log_2\left(\frac{30x}{6}\right) \rightarrow \log_2 5x$

b. $2 \log_3(x) + \log_3(5)$ $\xrightarrow{\text{power}} \log_3 x^2 + \log_3 5 \xrightarrow{\text{product}} \log_3 5x^2$

c. $\log_7(3x-2) = 2 \rightarrow 7^2 = 3x-2 \rightarrow 49 = 3x-2 \rightarrow 47 = 3x \rightarrow x = \frac{47}{3}$

d. $\log(2x+1) = -1 \rightarrow 10^{-1} = 2x+1 \rightarrow \frac{1}{10} = 2x+1 \rightarrow 2x = \frac{-9}{10} \rightarrow x = \frac{-9}{20}$

e. $\log_5(3y) + \log_5(9) = \log_5(405) \rightarrow \log_5 27y = \log_5 405 \xrightarrow{\text{drop the log}} 27y = 405 \rightarrow y = 15$

f. $\log(x) + \log(x+21) = 2 \rightarrow \log(x^2+21x) = 2 \rightarrow 10^2 = x^2+21x \rightarrow x^2+21x-100=0$

$x=4$ only \leftarrow $x=25, x=4 \leftarrow (x+25)(x-4)=0$

10. Find the equation of the exponential function that passes through the points (2, 264) and (6, 4044) and has a horizontal asymptote at $y = 12$. (hint: think about 6.2.3 and use $y = ab^x + c$.)

$$\begin{matrix} & a & b & c \\ C = 12 & & & \end{matrix}$$

$$264 = a \cdot b^2 + 12$$

$$4044 = a \cdot b^6 + 12$$

$$252 = a \cdot b^2$$

$$4032 = a \cdot b^6$$

$$\frac{252}{b^2} = a$$

$$4032 = \frac{252}{b^2} b^6$$

$$2 = b$$

$$4032 = 252 b^4$$

$$\frac{4032}{252} \rightarrow 16 = b^4$$

$$63 = a$$

$$\boxed{y = 63 \cdot 2^x + 12}$$

11. Find the inverse of each of the functions below. Write your answers in function notation.

a.) $p(x) = \frac{1}{2}(x+4)^2 + 1$

$$y = \frac{1}{2}(x+4)^2 + 1$$

$$x = \frac{1}{2}(y+4)^2 + 1$$

$$x-1 = \frac{1}{2}(y+4)^2$$

$$2(x-1) = (y+4)^2$$

$$\sqrt{2(x-1)} = y+4$$

$$\sqrt{2(x-1)} - 4 = y$$

$$\boxed{p^{-1}(x) = -4 + \sqrt{2(x-1)}}$$

b.) $k(x) = 3\log_2(x-2)$

$$y = 3 \cdot \log_2(x-2)$$

$$x = 3 \cdot \log_2(y-2)$$

$$\frac{x}{3} = \log_2(y-2)$$

$$2^{\frac{x}{3}} = y-2$$

$$\boxed{k^{-1}(x) = -2 + 2^{\frac{x}{3}}}$$

12. Define the vocabulary. You may use words, examples, pictures, symbols to convey what you mean.

Logarithm:

Power property of logs:

Asymptote:

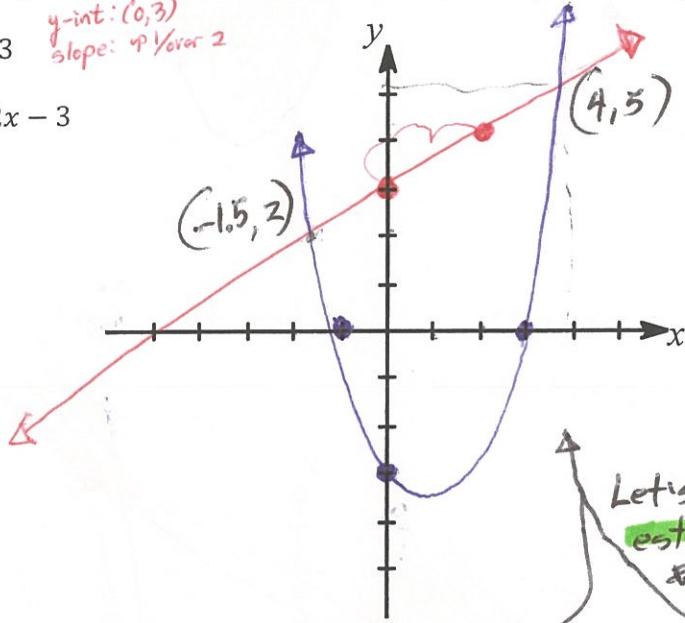
X-intercepts:

Plane (in the context of algebra 2):

13. Graph the system: (hint: one of them is a line, one is a parabola {extra credit if you complete the square to put it into $y = a(x - h)^2 + k$ })

a. $y = \frac{1}{2}x + 3$ *y-int: (0, 3)
slope: up 1 over 2*

b. $y = x^2 - 2x - 3$



b. Use the graph to solve $\frac{1}{2}x + 3 = x^2 - 2x - 3$.

only has x ... x = -1.5 and x = 4

$$y = x^2 - 2x - 3$$

$x\text{-int } y=0$ $0 = x^2 - 2x - 3$ $0 = (x-3)(x+1)$ $x = 3 \text{ or } x = -1$ $(3, 0) \quad (-1, 0)$	$y\text{-int } x=0$ $y = (0)^2 - 2(0) - 3$ $y = -3$ $(0, -3)$
---	--

c. Solve $y = \cancel{x^2 - 2x - 3}$

*has x and y y = ~~x^2 - 2x - 3~~ + 6
(-1.5, 2) and (4, 5)*

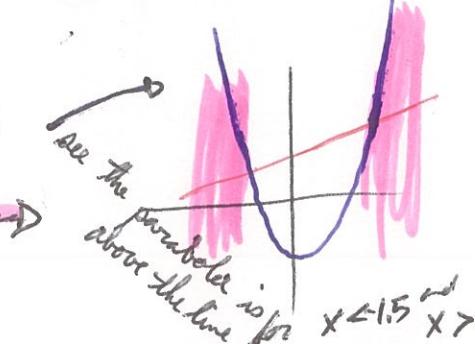
d. What is the difference between the answer to b and c? Explain why this is so.

b only cares about x ... c cares about x and y

e. Solve and show the solution on a number line: $\frac{1}{2}x + 3 < x^2 - 2x - 3$.

where is the line below the parabola " $<$ "

number line
-1.5

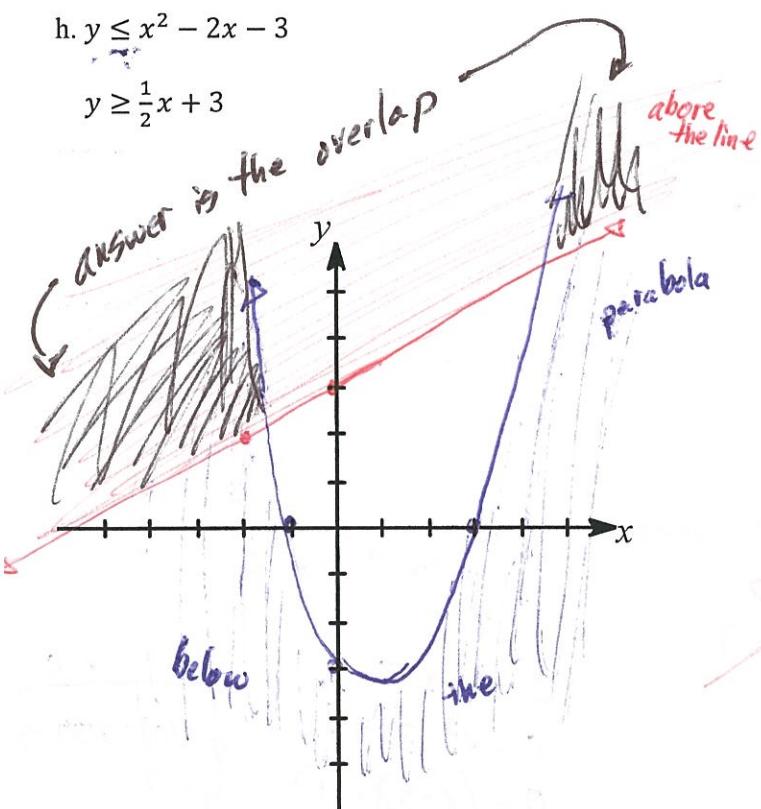


$\geq \rightarrow$ shade above
 $\leq \rightarrow$ shade below

For problems f and g, solve by shading the appropriate region.

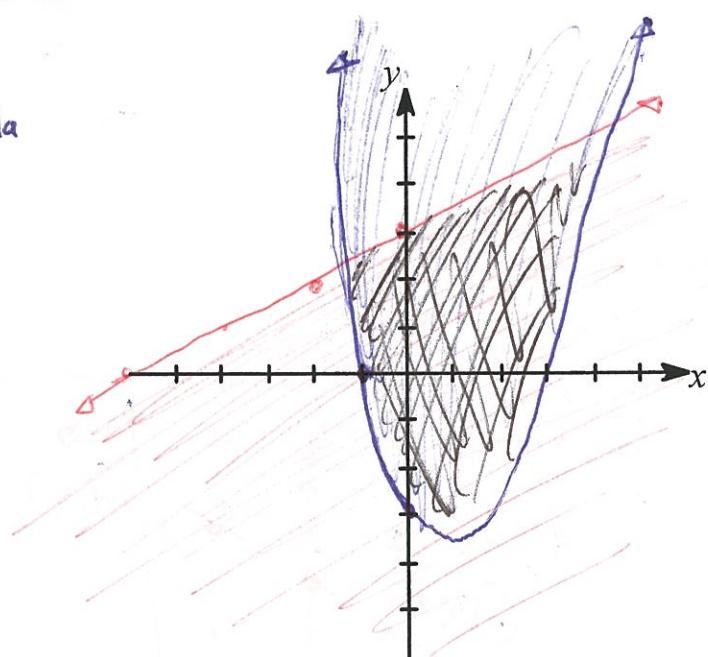
h. $y \leq x^2 - 2x - 3$

$y \geq \frac{1}{2}x + 3$



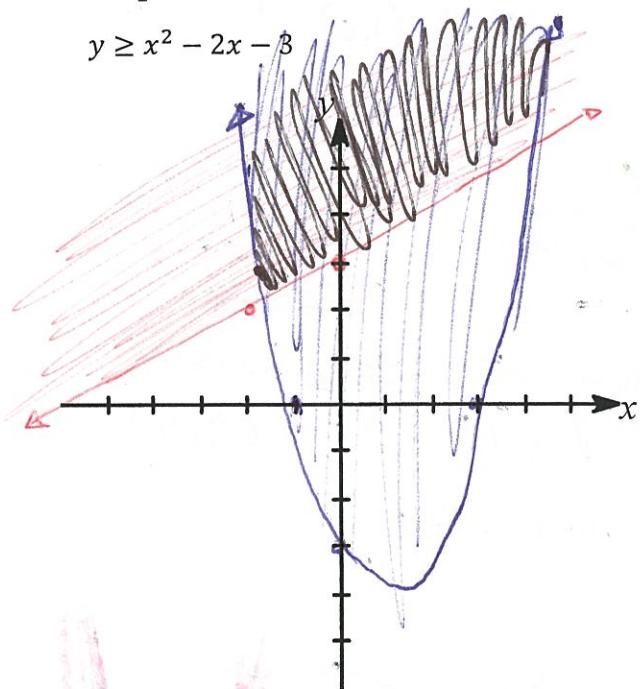
i. $y \leq \frac{1}{2}x + 3$

$y \leq x^2 - 2x - 3$



j. $y \geq \frac{1}{2}x + 3$

$y \geq x^2 - 2x - 3$



k. $y \geq \frac{1}{2}x + 3$

$y \leq x^2 - 2x - 3$

Same as h. Oppos

