

Chapter 6 Assessment. Complete these problems.

1.(2 points each) If $f(x) = 3x^4 - 7$ and $g(x) = -7(2x + 5)$, find the value of each expression below.

$$\begin{aligned} a.) f(-2) &= 3(-2)^4 - 7 \\ &= 3(16) - 7 \\ &= 48 - 7 \\ &= \boxed{41} \end{aligned}$$

$$\begin{aligned} b.) g(-2) &= -7(2(-2) + 5) \\ &= -7(-4 + 5) \\ &= -7(1) \\ &= \boxed{-7} \end{aligned}$$

$$\begin{aligned} c.) f(g(-3)) &= 3(-7(2(-3) + 5))^4 - 7 \\ &= 3(-7(-1))^4 - 7 \\ &= \boxed{7,196} \end{aligned}$$

$$\begin{aligned} d.) g(f(-3)) &= -7(2(3(-3)^4 - 7) + 5) \\ &= -7(2(81 - 7) + 5) \\ &= -7(145 + 5) \\ &= -7(150) = \boxed{-1,050} \end{aligned}$$

e.) Are $f(x)$ and $g(x)$ inverses of each other? Justify your answer.

No.

2(5 points). Simplify the following expressions:

$$a.) \left(\frac{7x^5}{2x^2}\right) \cdot \frac{x^6y^5}{343x^{11}} \cdot \frac{14x^6y^{12}}{x^9y^7}$$

$$\frac{7 \cdot 14 \cdot x^6 \cdot x^5 \cdot x^6 \cdot y^5 \cdot y^{12}}{2 \cdot 343 \cdot x^2 \cdot x^{11} \cdot x^9 \cdot y^7} = \frac{7 \cdot 7 \cdot 2 \cdot x^{17} \cdot y^{17}}{2 \cdot 7 \cdot 7 \cdot 7 \cdot x^{22} \cdot y^7} = \boxed{\frac{y^{10}}{7^2 x^5}}$$

3(5 points). Subtract and simplify the following expressions:

a.) $\frac{3x+5}{x+5} + \frac{x+1}{x-2} - \frac{4x^2-3x-1}{x^2+3x-10}$

$$\frac{(x-2)}{(x-2)} \frac{3x+5}{x+5} + \frac{x+1}{x-2} \cancel{\frac{(x+5)}}{(x+5)} - \frac{(4x+1)(x-1)}{(x-2)(x+5)}$$

$$\frac{3x^2-6x+10}{(x-2)(x-5)} + \frac{x^2+x+5x+5}{(x-2)(x-5)} - (4x^2-3x-1) \quad \leftarrow \text{apply mistake - ,}$$

$$\boxed{\frac{8x+4}{(x-2)(x-5)}}$$

$$4x^2-3x-1$$

$$4x^2-4x+x-1$$

$$4x(x-1)+1(x-1)$$

~~$$4x^2-4x+x-3$$~~

4(5 points). Identify the vertex and the line of symmetry: $f(x) = 3x^2 + 24x - 16$

vertex: $(-4, -64)$

los: $x = -4$

$$= 3(x^2 + 8x) - 16$$

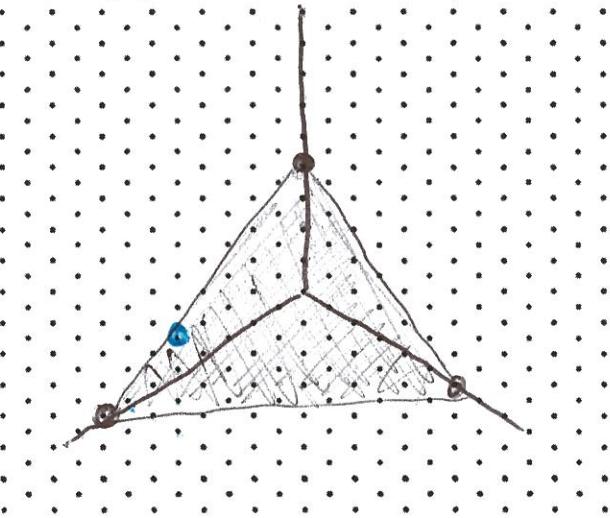
$$= 3(x^2 + 8x + 16) - 16 - 48$$

" + 48 "

$$= 3(x+4)^2 - 64$$

5. Graph on Isometric graph paper (part a is 7 points, part b is 3 points)

a. Sketch a graph of the solution to the equation: $3x + 4y + 6z = 24$



shape: +3

integers only +3

b. Graph the point $(7,2,3)$. What are three other points that have the same isometric dot position?

$$(8, 3, 4)$$

$$(6, 1, 2)$$

$$(5, 0, 1)$$

$$(9, 4, 5)$$

Algebra 2

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6(10 points). Solve for x , y , and z in the system of equations at below.

$$\begin{array}{l} \textcircled{1} -3x + 3y + 6z = 30 \\ \textcircled{2} 6x - 3y - z = -23 \\ \textcircled{3} -5x - 6y + 3z = -19 \end{array}$$

$$\begin{aligned} & \textcircled{5} \textcircled{1} - 6y + 3(\textcircled{2}) = -19 \\ & 5 - 6y + 6 = -19 \\ & -6y + 11 = -19 \\ & -6y = -30 \\ & y = 5 \end{aligned}$$

$$\begin{array}{r} \textcircled{1} -3x + 3y + 6z = 30 \\ \textcircled{2} 6x - 3y - z = -23 \\ \hline -3x + 5z = 7 \end{array}$$

$$\begin{array}{r} \textcircled{2} -6x + 6y + 12z = 60 \\ \textcircled{3} -5x - 6y + 3z = -19 \\ \hline -11x + 15z = 41 \end{array}$$

$$3(-1) + 5z = 7$$

$$5z = 10$$

$$z = 2$$

$$\begin{array}{r} \textcircled{4} -3 \\ \hline -9x - 15z = -21 \\ -20x = 20 \end{array}$$

$$x = -1$$

↑
sign error -2

only step 1 attempt 4

$$\boxed{(-1, 5, 2)}$$

7(3 points each). Solve each equation below for x .

$$\textcircled{a}.) 20 = 4^x$$

$$\begin{aligned} & \log 20 = \log 4^x \\ & \log 20 = x \log 4 \\ & \frac{\log 20}{\log 4} = x \end{aligned}$$

$$\begin{array}{r} 1.301 \\ 0.602 \\ \hline 2.161 = x \end{array}$$

$$\textcircled{b}.) 2 \cdot 6^x = 40$$

$$\begin{aligned} & 6^x = 20 \\ & \log 6^x = \log 20 \\ & x \cdot \log 6 = \log 20 \\ & x = \frac{\log 20}{\log 6} \\ & x = \frac{1.301}{0.778} = \boxed{1.693} \end{aligned}$$

$$\textcircled{c}.) 3 \cdot 5^{2x-3} = 72$$

$$5^{2x-3} = 24$$

$$(2x-3) \log 5 = \log 24$$

$$2x-3 = \frac{\log 24}{\log 5}$$

$$2x = 3 + \frac{\log 24}{\log 5}$$

$$\begin{aligned} x &= 3 + \frac{\log 24}{\log 5} = \frac{3 + \frac{1.3802}{0.6989}}{2} = \boxed{2.487} \end{aligned}$$

8(10 points). What is the equation of the parabola passing through the points (1, 1), (2, 6), and (3, 17) ?

$$\begin{aligned} 1 &= a(1)^2 + b(1) + c \rightarrow 1 = a + b + c \\ 6 &= a(2)^2 + b(2) + c \rightarrow 6 = 4a + 2b + c \\ 17 &= a(3)^2 + b(3) + c \rightarrow 17 = 9a + 3b + c \end{aligned}$$

$$1 = a + b + c$$

$$1 = 3 + 4 + c$$

$$2 = c$$

$$\begin{array}{l} \textcircled{3} \quad 17 = 9a + 3b + c \\ -\textcircled{1} \quad -1 = -a - b - c \\ \hline 16 = 8a + 2b \quad \textcircled{+1} \\ -10 = -6a - 2b \quad \xleftarrow{x-2} \\ \textcircled{+2} \quad 6 = 2a \\ 3 = a \end{array} \quad \begin{array}{l} \textcircled{2} \quad 6 = 4a + 2b + c \\ -\textcircled{1} \quad -1 = -a - b - c \\ \hline 5 = 3a + b \\ 5 = 3(3) + b \\ -4 = b \end{array}$$

$$y = 3x^2 - 4x + 2$$

9(2 points each). In parts (a) through (h), rewrite each expression as a single logarithm.

Write as a single logarithm

a.) $\log_3(7) + \log_3(y)$

$$\boxed{\log_3 7y}$$

b.) $\log_2(d) - \log_2(x)$

$$\boxed{\log_2 \frac{d}{x}}$$

c.) $\log_6(r) + 3\log_6(x)$

$$\boxed{\log_6 rx^3}$$

d.) $\log(12) + \log(5) - \log(2)$

$$\log \frac{60}{2} = \boxed{\log 30}$$

e.) $\log_4(16x) - \log_4(x)$

$$\boxed{\log_4 16} \circ 2$$

f.) $\log_5(\sqrt[3]{x^5}) + \log_5(\sqrt[4]{x^3})$

$$\log_5 x^{\frac{5}{3}} \cdot x^{\frac{3}{4}} = \log_5 x^{\frac{29}{12}}$$

g.) $\log_7(4) + \log_3(2)$

Not possible

h.) $\log_5(x-1) + \log_5(x+1)$

$$\log_5(x-1)(x+1) = \boxed{\log_5(x^2-1)}$$

i.) $2\log_{11}(4) + 4\log_{11}(2) - 3\log_{11}3$

$$\log_{11}4^2 + \log_{11}2^4 - \log_{11}3^3$$

$$\log_{11}16 + \log_{11}16 - \log_{11}27$$

$$\log_{11} \frac{256}{27}$$

j.) $\log_2(2x-7) + \log_2(5x+9)$

$$\log_2(2x-7)(5x+9)$$

$$\log_2(10x^2 - 17x - 63)$$

$$\begin{array}{r} -35x \\ 18x \\ \hline -17x \end{array}$$

10.(3 points each) Solve each logarithmic equation for x :

a.) $\log_{16}(3x - 8) = \frac{1}{2}$

$$\begin{aligned} 16^{\frac{1}{2}} &= 3x - 8 \\ 4 &= 3x - 8 \\ 12 &= 3x \\ 4 &= x \end{aligned}$$

b.) $\log_7(4x + 1) = 2$

$$\begin{aligned} 7^2 &= 4x + 1 \\ 49 &= 4x + 1 \\ 48 &= 4x \\ 12 &= x \end{aligned}$$

c.) $\log_6(6^{72}) = x$

$$72 = x$$

d.) $2^{\log_2(2x-9)} = y$

$$\begin{aligned} 2x - 9 &= y \\ 2x &= y + 9 \\ x &= \frac{y+9}{2} \end{aligned}$$

e.) $\log_4(2x) = 4\log_4(3) + \log_4(7)$

$$\log_4 2x = \log_4 81 + \log_4 7$$

$$\log_4 2x = \log_4 567$$

$$2x = 567$$

$$x = 283.5$$

f.) $\log_2(x^2 + 2x) = 3$

$$2^3 = x^2 + 2x$$

$$0 = x^2 + 2x - 8$$

$$0 = (x+4)(x-2)$$

$$x = -4 \text{ or } x = 2$$

g.) $\log_7(x+1) + \log_7(x-5) = 1$

$$\log_7(x+1)(x-5) = 1$$

$$7^1 = x^2 - 5x + x - 5$$

$$0 = x^2 - 4x - 12$$

$$0 = (x-6)(x+2)$$

$$\begin{aligned} x &= 6 \quad \text{or} \quad x = -2 \\ &\uparrow \quad \uparrow \\ &\text{extraneous} \quad \log_7(-2+1) \\ &\uparrow \quad \uparrow \\ &\text{impossible} \end{aligned}$$

11(10 points). Find the equation of the exponential function that passes through the points (2, 13) and (5, 209) and has a horizontal asymptote at $y = -15$. (hint: think about 6.2.3 and use $y = ab^x + c$. Also sketch the graph...)

$$\begin{aligned}
 13 &= ab^2 - 15 & 209 &= ab^5 - 15 & \text{set up} \\
 28 &= ab^2 & 224 &= ab^5 \\
 \frac{28}{b^2} &= a & 224 &= \frac{28}{b^2} \cdot b^5 \\
 \frac{28}{(b^2)^2} &= a & 224 &= \frac{28b^3}{b^2} \\
 \frac{28}{b^4} &= a & 8 &= b^3 \\
 \frac{28}{4} &= a & 2 &= b \\
 7 &= a
 \end{aligned}$$

$$y = 7 \cdot 2^x - 15$$

12.) Consider the following expression: $\log_3 67$

a.(3 points) Estimate the value of this logarithm and *explain* why your estimate makes sense.

$$3^3 = 27 \quad 3^4 = 81 \quad \text{between 3 and 4}$$

b.(2 points) Write the logarithm as an exponent, use x as the right side of the equation.

$$\log_3 67 = x \quad \dots \quad 3^x = 67$$

c.(5 points) Evaluate the logarithm showing all work. Round your answer to the thousandths place.

$$\begin{aligned}
 3^x &= 67 \\
 \log 3^x &= \log 67 \\
 x \cdot \log 3 &= \log 67 \\
 x &= \frac{\log 67}{\log 3} = \frac{1.826}{0.4771} = 3.827
 \end{aligned}$$

Algebra 2

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13(5 points each.) Find the inverse of each of the functions below. Write your answers in function notation.

Name:

Period:

a.) $p(x) = \frac{2}{3}(2x - 7)^3 + 21$

$$X = \frac{2}{3}(2y - 7)^3 + 21$$

$$X - 21 = \frac{2}{3}(2y - 7)^3$$

$$\frac{3}{2}(X - 21) = (2y - 7)^3$$

$$\sqrt[3]{\frac{3}{2}(X - 21)} = 2y - 7$$

$$7 + \sqrt[3]{\frac{3}{2}(X - 21)} = 2y$$

$$\frac{7 + \sqrt[3]{\frac{3}{2}(X - 21)}}{2} = y$$

b.) $k(x) = 3\log_2(x - 2)$

$$y = 3\log_2(x - 2)$$

$$x = 3\log_2(y - 2)$$

$$\frac{x}{3} = \log_2(y - 2)$$

$$2^{\left(\frac{x}{3}\right)} = y - 2$$

$$2 + 2^{\left(\frac{x}{3}\right)} = k^{-1}(x)$$

14(2points each). Define the vocabulary. You may use words, examples, pictures, symbols to convey what you mean.

Logarithm:

Power property of logs:

Asymptote:

X-intercepts:

Plane (in the context of algebra 2):

(C)

(O)

(C)