

1. If $f(x) = x^4$ and $g(x) = 3(x + 2)$, find the value of each expression below.

a. $f(2)$

$$\begin{aligned} f(2) &= (2)^4 \\ &= \boxed{16} \end{aligned}$$

b. $g(2) = 3((2) + 2)$

$$\begin{aligned} &= 3(4) \\ &= \boxed{12} \end{aligned}$$

12^4

c. $f(g(2)) = (3(2+2))^4$

$$\begin{aligned} &= (12)^4 \\ &= \boxed{20736} \end{aligned}$$

d. $g(f(2)) = 3((16) + 2)$

$$= \boxed{54}$$

e. Are $f(x)$ and $g(x)$ inverses of each other? Justify your answer.

No, $|f(g(x)) = x \Rightarrow$ means inverse | $f(g(2)) = 20736$
 ↗ counterexample

$$\log_{16} 16 = 1$$

$$16^x = 16$$

$$16^{-1} = \frac{1}{16}$$

$$\log_{16} 16 = x$$

$$16^{-2} = \frac{1}{16^2}$$

2. Add or subtract each pair of rational expressions. Simplify the result.

a. $\frac{4}{x^2+5x+6} + \frac{2x}{x+2}$

*Adding rationals
↳ fractions*

$\frac{1}{6} + \frac{1}{3}$

$\frac{1}{3 \cdot 2} + \frac{1}{3} \cdot \frac{2}{2}$

$\frac{3}{6}$

$\frac{4}{(x+3)(x+2)} + \frac{2x^2+6x}{(x+2)(x+3)}$

$\frac{1}{6} + \frac{2}{6}$

$\frac{3}{6}$

$\frac{2x^2+6x+4}{(x+3)(x+2)} = \frac{2(x^2+3x+2)}{(x+3)(x+2)} = \frac{2(x+2)(x+1)}{(x+3)(x+2)}$

$= \boxed{\frac{2(x+1)}{(x+3)}}$

b. $\frac{3x^2+x}{(2x+1)^2} - \frac{3}{2x+1}$

$$\frac{1}{2x+2} + \frac{1}{2}$$

$$\frac{3x^2+x}{(2x+1)(2x+1)} - \frac{3}{2x+1} \cdot \frac{2x+1}{2x+1}$$

$$\frac{3x^2+x}{(2x+1)(2x+1)} - \frac{6x+3}{(2x+1)(2x+1)}$$

~~$$\begin{array}{r} -9x^2 \\ -5x \end{array}$$~~

1.9
3.5

$$\frac{3x^2+x-6x-3}{(2x+1)(2x+1)} = \left[\frac{3x^2-5x-3}{(2x+1)(2x+1)} \right] = \frac{3x^2-5x-3}{4x^2+4x+1}$$

3. Find the inverse of each of the functions below. Write your answers in function notation.

a. $p(x) = 3(x^3 + 6)$

$$\begin{aligned} y &= 3(x^3 + 6) \\ x &= 3(y^3 + 6) \\ \frac{x}{3} &= y^3 + 6 \\ -6 & \\ \sqrt[3]{\frac{x}{3}-6} &= y \\ \sqrt[3]{\frac{x}{3}-6} &= y \end{aligned}$$

$$p(x) = 3(x^3 + 6)$$

- 1.) cube it
 - 2.) add 6
 - 3.) mult. by 3
- 1.) divide by 3
 - 2.) sub 6
 - 3.) cube root

$$p^{-1}(x) = \sqrt[3]{\frac{x}{3}-6}$$

b. $k(x) = 3x^3 + 6$

$$\begin{aligned} y &= 3x^3 + 6 \\ x &= 3y^3 + 6 \\ -6 & \\ \frac{x-6}{3} &= y^3 \\ \sqrt[3]{\frac{x-6}{3}} &= y \\ \sqrt[3]{\frac{x-6}{3}} &= y \end{aligned}$$

$$k(x) = 3x^3 + 6$$

- 1.) cube it
 - 2.) times 3
 - 3.) add 6
- 1.) subtract 6
 - 2.) divide by 3
 - 3.) cube root

$$k^{-1}(x) = \sqrt[3]{\frac{x-6}{3}}$$

c. $h(x) = \frac{x+1}{x-1}$

d. $j(x) = \frac{2}{3-x}$

4. Solve for x without using a calculator.

a. $x = \log_{25}(5)$

b. $\log_x(1) = 0$

c. $23 = \log_{10}(x)$

5. Find the equation of the parabola that passes through the points $(-2, 24)$, $(3, -1)$, and $(-1, 15)$.

$$\begin{aligned}
 & y = ax^2 + bx + c \\
 & 24 = a(-2)^2 + b(-2) + c \quad \rightarrow \textcircled{1} \quad 24 = 4a - 2b + c \\
 & -1 = a(3)^2 + b(3) + c \quad \rightarrow \textcircled{2} \quad -1 = 9a + 3b + c \\
 & 15 = a(-1)^2 + b(-1) + c \quad \rightarrow \textcircled{3} \quad 15 = a - b + c
 \end{aligned}$$

$$\begin{array}{rcl}
 \textcircled{2} - \textcircled{1} & -1 = 9a + 3b + c & \textcircled{1} \quad 24 = 4a - 2b + c \\
 -\textcircled{3} & -15 = -a + b - c & -\textcircled{2} \quad 1 = -9a - 3b - c \\
 \hline
 -16 & 8a + 4b & 25 = -5a - 5b \\
 & \cancel{\div 4} & \cancel{\div 5} \\
 \textcircled{1} & -4 = 2a + b & \textcircled{4} \quad 5 = -a - b
 \end{array}$$

$$\begin{array}{rcl}
 \textcircled{11} & -4 = 2a + b & -4 = 2(1) + b \\
 \textcircled{12} & 5 = -a - b & -4 = 2 + b \\
 \hline
 & 1 = a & -6 = b
 \end{array}$$

$$\begin{aligned}
 15 &= a - b + c \\
 15 &= (1) - (-6) + c \\
 15 &= 7 + c \\
 8 &= c
 \end{aligned}$$

$$y = 1x^2 - 6x + 8$$

6. Solve each of the following equations to the nearest 0.001.

a. $(5.825)^{x-3} = 120$

$$\log 5.825^{x-3} = \log 120$$

$$(x-3) \log 5.825 = \log 120$$

$$x-3 = \frac{\log 120}{\log 5.825}$$

$$x = 3 + \frac{\log 120}{\log 5.825}$$

b. $18(1.2)^{2x-1} = 900$

divide by 18
log both sides
power property
divide by log 1.2
add 1
divide by 2

7. Identify the vertex and the line of symmetry for each one.

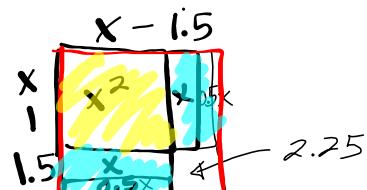
a. $f(x) = 4x^2 - 12x + 6$

$$f(x) = 4(x^2 - 3x) + 6$$

$$f(x) = 4(x^2 - 3x + 2.25) + 6 - 9$$

$$f(x) = 4(x - 1.5)^2 - 3$$

vertex: $(1.5, -3)$



to keep the balance
 $4(2.25) = 9$

b. $g(x) = 2x^2 + 14x + 4$

$$2(x^2 + 7x + 12.25) + 4 - 27.5$$

$$2(x + 3.5)^2 - 20.5$$

vertex: $(-3.5, -20.5)$

8. Solve each system of equations below.

a. $-4x = z - 2y + 12$

$y + z = 12 - x$

$8x - 3y + 4z = 1$

b. $3x + y - 2z = 6$

$x + 2y + z = 7$

$6x + 2y - 4z = 12$

9. Multiply or divide each pair of rational expressions. Simplify the result.

a. $\frac{x^2 - 16}{(x-4)^2} \cdot \frac{x^2 - 3x - 18}{x^2 - 2x - 24}$

$$\frac{(x-4)(x+4)}{(x-4)(x-4)} \cdot \frac{(x-6)(x+3)}{(x-6)(x+4)}$$

$$\frac{(x+4)(x+3)}{(x-4)(x+4)}$$

$$\boxed{\frac{(x+3)}{(x-4)}}$$

b. $\frac{x^2 - 1}{x^2 - 6x - 7} \div \frac{x^3 + x^2 - 2x}{x-7}$

10. Find the equation of an exponential function that passes through the points $(2, 48)$ and $(5, 750)$.

Definition of logs: $\log_m(a) = n$ means $m^n = a$

Product Property: $\log_m(a \cdot b) = \log_m(a) + \log_m(b)$

Quotient Property: $\log_m\left(\frac{a}{b}\right) = \log_m(a) - \log_m(b)$

Power Property: $\log_m(a^n) = n \cdot \log_m(a)$

Inverse relationship: $\log_m(m)^n = n$ and $m^{\log_m(n)} = n$

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Inverse relationship: $\log_m(m)^n = n$ and $m^{\log_m(n)} = n$

$$\frac{2 \cdot x}{2} = x \quad x + 1 - 1 = x$$

$$\sqrt[2]{x^2} = x \quad 2^{\log_2 x} = x$$

$$\sqrt[3]{x^3} = x \quad \log_2 2^x = x$$

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1. $\log_3(5) + \log_3(m)$

$$\boxed{\log_3(5m)}$$
 product property of logs

2. $\log_2(q) - \log_2(z)$

$$\boxed{\log_2\left(\frac{q}{z}\right)}$$
 quotient

3. $\log_6(r) + 3\log_6(x)$

$$\boxed{\log_6 r + \log_6 x^3}$$
 power

$$\log_6 r \cdot x^3$$

4. $\log(90) + \log(4) - \log(36)$

$$\log 360 - \log 36$$
 product

$$\boxed{1}$$
 quotient

5. $\log_4(16x) - \log_4(x)$

$$\log_4\left(\frac{16x}{x}\right)$$

$$\log_4 16$$

6. $\log(\sqrt{x}) + \log(x^2)$

$$\log(x^{\frac{1}{2}}) + \log(x^2)$$

$$\log(x^{\frac{1}{2}+2})$$

$$\boxed{\log(x^{\frac{5}{2}})}$$

7. $\log_5(\sqrt{x}) + \log_5(\sqrt[3]{x})$

$$\log_5(x^{\frac{1}{2}}) + \log_5(x^{\frac{1}{3}})$$

$$\log_5(x^{\frac{1}{2}}x^{\frac{1}{3}})$$

$$\log_5(x^{\frac{1}{2}+\frac{1}{3}})$$

$$\boxed{\log_5(x^{\frac{5}{6}})}$$

$$\frac{1}{2} + \frac{1}{3}$$

$$\frac{3}{6} + \frac{2}{6}$$

$$\frac{5}{6}$$

8. $\log_5(x-1) + \log_5(x+1)$

$$\log_5(x-1)(x+1)$$

$$\log_5(x^2 - x + x - 1)$$

$$\boxed{\log_5(x^2 - 1)}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\left(\sqrt[4]{x^3}\right) \times \left(\sqrt[3]{x^2}\right)$$

$$(x^{\frac{3}{4}})(x^{\frac{2}{3}})$$

$$x^{\frac{3}{4} + \frac{2}{3}}$$

$$x^{\frac{17}{12}}$$

$$\frac{3}{4} + \frac{2}{3}$$

$$\frac{9}{12} + \frac{8}{12}$$

$$\frac{17}{12}$$

$$\log_2 x + \log_3 y \quad \text{cannot}$$

$$\log_2 x + 3 \log_3 y \quad \text{can but just part}$$

$$\log_2 x + \log_3 y^3$$

Simplify
#84.) $3 \log_2 x - 7 \log_3 p + 8 \log_2 y - 3 \log_3 w$

$$\log_2 x^3 - \log_3 p^7 + \log_2 y^8 - \log_3 w^3$$

$$\log_2 x^3 + \log_2 y^8 - \underbrace{\log_3 p^7 - \log_3 w^3}_{\text{ }}$$

$$\log_2 x^3 y^8 - \log_3 \left(\frac{p^7}{w^3} \right)$$

9. $\log_4(2x+3) = \frac{1}{2}$ we inverse property
 convert to exponential
 $\log_4(2x+3) = \frac{1}{2}$
 $4^{\frac{1}{2}} = 2x+3$
 $2 = 2x+3$
 $-1 = 2x$
 $\frac{-1}{2} = x$

10. $\log_5(3x+1) = 2$
 $5^2 = 3x+1$
 $25 = 3x+1$
 $-1 = -1$
 $24 = 3x$
 $x = 8$

10. $\log_5(3x+1) = 2$

$$\begin{aligned} 5^2 &= 3x+1 \\ 25 &= 3x+1 \\ -1 &= -1 \\ 24 &= 3x \end{aligned}$$

$$x = 8$$

11. $\log_9(9^2) = x$
 $\log_9(9^2) = 2$

$$x = 2$$

12. $16^{\log_{16}(5)} = y$

$$5 = y$$

13. $8^{\log_8(x)} = 3$

$$x = 3$$

14. $\log_5(5^{0.3}) = y$

$$0.3 = y$$

15. $\log_2(x) = 3\log_2(4) + \log_2(5)$

$$\log_2 x = \log_2 4^3 + \log_2 5$$

16. $\log_6(x) + \log_6(8) = \log_6(48)$

$$\log_6(8x) = \log_6(48)$$

$$\log_2 x = \log_2 64 + \log_2 5$$

$$2 \log_2 x = \log_2 320$$

$$X = 320$$

$$\frac{8}{8} x = \frac{48}{8}$$

$$x = 6$$

$$4^{2x} = 16^{3x+1}$$

$$(2^2)^{2x} = (2^4)^{3x+1}$$

$$\log_2 2^{4x} \quad \log_2 2^{12x+4}$$

power prop of exp $(x^a)^b = x^{ab}$

$$4x = 12x + 4$$

$$-8x = 4$$

$$x = -\frac{1}{2}$$

17. $\log_2(144) - \log_2(x) = \log_2(9)$

$$2 \log_2 \frac{144}{x} = \log_2 9$$

$$\cancel{x} \cdot \frac{144}{\cancel{x}} = 9 \cdot \cancel{x}$$

$$9x = 144$$

$$x = 16$$

18. $\log_2(36) - \log_2(y) = \log_2(12)$

$$\log_2 \frac{36}{y} = \log_2 12$$

$$\frac{36}{y} = 12$$

$$36 = 12y$$

$$3 = y$$

19. $\log_5(3x-1) = -1$

convert to exponent

$5^{-1} = 3x-1$

$\frac{1}{5} = 3x-1$

$\frac{6}{5} = 3x$

$\frac{6}{15} = x$

$$\begin{array}{l} 5^{\text{both sides}} \\ 5^{\log_5(3x-1)} = 5^{-1} \end{array}$$

$3x-1 = 5^{-1}$

$$\begin{array}{l} \vdots \\ x = \frac{6}{15} \end{array}$$

20. $\log_2(x) - \log_2(3) = 4$

$\log_2 \frac{x}{3} = 4$

$2^4 = \frac{x}{3}$

$16 = \frac{x}{3}$

$\underline{48 = x}$

21. $\frac{1}{3} \log(3x+1) = 2$

$$\begin{array}{l} \log(3x+1)^{\frac{1}{3}} = 2 \\ (10^2)^3 = ((3x+1)^{\frac{1}{3}})^3 \\ 10^6 = (3x+1) \end{array}$$

$\frac{1000000-1}{3} = x$

$\frac{999999}{3} = x$

$\boxed{333,333 = x}$

22. $\log_2(5) + \frac{1}{2} \log_2(x) = \log_2(15)$

$$\begin{array}{l} \log_2 5 + \log_2 x^{\frac{1}{2}} = \log_2 15 \\ \log_2 5\sqrt{x} = \log_2 15 \\ 5\sqrt{x} = 15 \\ \sqrt{x} = 3 \\ x = 9 \end{array}$$

23. $\frac{1}{2} \log(y) = 2 \log(2) + \log(16)$

$$\log y^{\frac{1}{2}} = \log 2^2 + \log 16$$

$$\log y^{\frac{1}{2}} = \log 64$$

$$y^{\frac{1}{2}} = 64$$

$$\boxed{y = 1024}$$

24. $\log_2(x^2 + 2x) = 3$

$$2^3 = x^2 + 2x$$

$$0 = x^2 + 2x - 8$$

$$0 = (x+4)(x-2)$$

$$\begin{array}{c} x = -4 \text{ or } x = 2 \\ \checkmark \qquad \checkmark \end{array}$$

25. $2 \log_4(x) - \log_4(3) = 2$

$$\log_4 \frac{x}{3} = 2$$

$$16 = \frac{x}{3}$$

$$48 = x$$

26. $\log_7(x+1) + \log_7(x-5) = 1$

$$\log_7 (x+1)(x-5) = 1$$

$$\log_7 x^2 - 4x - 5 = 1$$

$$\log_7 (6+1)(6-5) = 1$$

$$\log_7 7 = 1$$

$$7^1 = x^2 - 4x - 5$$

$$0 = x^2 - 4x - 12$$

$$0 = (x-6)(x+2)$$

$$\log_7 (-2+1)(-2-5) = 1 \quad \boxed{x=6} \quad x = -2$$

$$\log_7 (-1)(-7) = 1$$

$$\log_7 7 = 1$$