17. First, write the equation in the form $y = a(x - h)^2 + k$.

$$(x-1)^2 = 12(y-1)$$
$$\frac{1}{12}(x-1)^2 = y - 1$$
$$\frac{1}{12}(x-1)^2 + 1 = y$$

Then use the following information to draw the graph.

vertex: $(1, 1)$

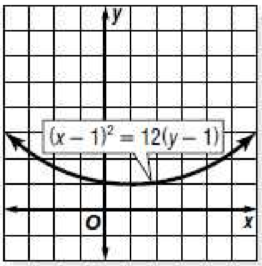
focus: $\left(1, 1 + \frac{1}{4\left(\frac{1}{12}\right)}\right)$ or $(1, 4)$

axis of symmetry: $x = 1$

directrix: $y = 1 - \frac{1}{4\left(\frac{1}{12}\right)}$ or $y = -2$

direction of opening: upward, since $a > 0$

length of latus rectum: $\left|\frac{1}{\frac{1}{12}}\right|$ or 12 units



18. First, write the equation in the form $y = a(x - h)^2 + k$.

$$y + 6 = 16(x-3)^2$$
$$y = 16(x-3)^2 - 6$$

Then use the following information to draw the graph.

vertex: $(3, -6)$

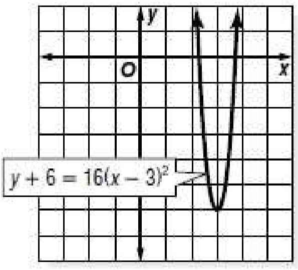
focus: $\left(3, -6 + \frac{1}{4(16)}\right)$ or $\left(3, -5\frac{63}{64}\right)$

axis of symmetry: $x = 3$

directrix: $y = -6 - \frac{1}{4(16)}$ or $y = -6\frac{1}{64}$

direction of opening: upward, since $a > 0$

length of latus rectum: $\left|\frac{1}{16}\right|$ or $\frac{1}{16}$ unit



19. First, write the equation in the form $y = a(x - h)^2 + k$.

$$x^2 - 8x + 8y + 32 = 0$$
$$(x^2 - 8x + \square) + 8y = -32 + \square$$
$$(x^2 - 8x + 16) + 8y = -32 + 16$$
$$(x-4)^2 + 8y = -16$$
$$8y = -(x-4)^2 - 16$$
$$y = -\frac{1}{8}(x-4)^2 - 2$$

Then use the following information to draw the graph.

vertex: $(4, -2)$

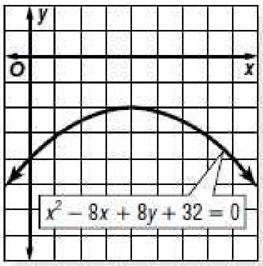
focus: $\left(4, -2 + \frac{1}{4\left(-\frac{1}{8}\right)}\right)$ or $(4, -4)$

axis of symmetry: $x = 4$

directrix: $y = -2 - \frac{1}{4\left(-\frac{1}{8}\right)}$ or $y = 0$

direction of opening: downward, since $a < 0$

length of latus rectum: $\left|\frac{1}{-\frac{1}{8}}\right|$ or 8 units



20. The equation is of the form $x = a(y - k)^2 + h$. Use the following information to draw the graph.

vertex: $(0, 0)$

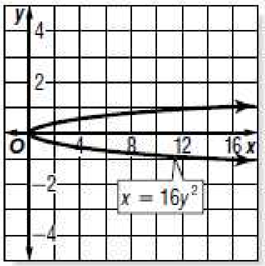
focus: $\left(0 + \frac{1}{4(16)}, 0\right)$ or $\left(\frac{1}{64}, 0\right)$

axis of symmetry: $y = 0$

directrix: $x = 0 - \frac{1}{4(16)}$ or $x = -\frac{1}{64}$

direction of opening: right, since $a > 0$

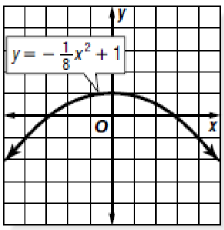
length of latus rectum: $\left|\frac{1}{16}\right|$ or $\frac{1}{16}$ unit



21. The vertex is at (0, 1), so $h = 0$ and $k = 1$. The focus is at (0, −1), so $k + \frac{1}{4a} = -1$. Use k to determine the value of a.

$$k + \frac{1}{4a} = -1$$
$$1 + \frac{1}{4a} = -1$$
$$\frac{1}{4a} = -2$$
$$1 = -8a$$
$$-\frac{1}{8} = a$$

An equation of the parabola is $y = -\frac{1}{8}(x - 0)^2 + 1$ or $y = -\frac{1}{8}x^2 + 1$.



22. The center of the circle is at (2, −3) and its radius is 5 units.

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 2)^2 + [y - (-3)]^2 = 5^2$$
$$(x - 2)^2 + (y + 3)^2 = 25$$

An equation for the circle is $(x - 2)^2 + (y + 3)^2 = 25$.

23. The center of the circle is at (−4, 0) and its radius is $\frac{3}{4}$ unit.

$$(x - h)^2 + (y - k)^2 = r^2$$
$$[x - (-4)]^2 + (y - 0)^2 = \left(\frac{3}{4}\right)^2$$
$$(x + 4)^2 + y^2 = \frac{9}{16}$$

An equation for the circle is $(x + 4)^2 + y^2 = \frac{9}{16}$.

24. The center of the circle is the midpoint of the diameter.

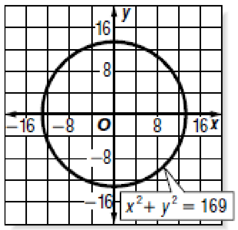
$$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{9 + (-3)}{2}, \frac{4 + (-2)}{2}\right)$$
$$= \left(\frac{6}{2}, \frac{2}{2}\right)$$
$$= (3, 1)$$

The radius is the distance from the center to one of the points.

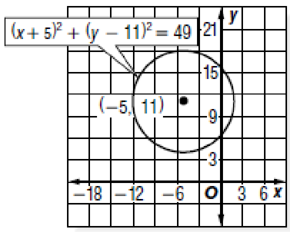
$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(9 - 3)^2 + (4 - 1)^2}$$
$$= \sqrt{6^2 + 3^2}$$
$$= \sqrt{36 + 9}$$
$$= 3\sqrt{5}$$

25. Since the circle is tangent to the x-axis, its radius is 2. Since $r = 2$, $r^2 = 4$. The center is at (−1, 2), so $h = -1$ and $k = 2$. Substitute h, k, and r^2 into the standard form of the equation of a circle. An equation of the circle is $[x - (-1)]^2 + (y - 2)^2 = 4$ or $(x + 1)^2 + (y - 2)^2 = 4$.

26. The center of the circle is at (0, 0) and the radius is $\sqrt{169}$ or 13 units.

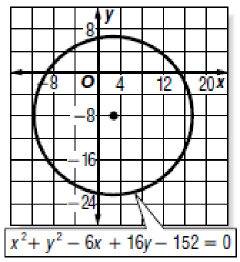


27. The center of the circle is at (−5, 11) and the radius is $\sqrt{49}$ or 7 units.



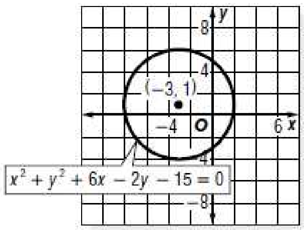
28.
$$x^2 + y^2 - 6x + 16y - 152 = 0$$
$$(x^2 - 6x + \square) + (y^2 + 16y + \square) = 152 + \square + \square$$
$$(x^2 - 6x + 9) + (y^2 + 16y + 64) = 152 + 9 + 64$$
$$(x - 3)^2 + (y + 8)^2 = 225$$

The center of the circle is at (3, −8) and the radius is $\sqrt{225}$ or 15 units.



29.
$$x^2 + y^2 + 6x - 2y - 15 = 0$$
$$(x^2 + 6x + \square) + (y^2 - 2y + \square) = 15 + \square + \square$$
$$(x^2 + 6x + 9) + (y^2 - 2y + 1) = 15 + 9 + 1$$
$$(x + 3)^2 + (y - 1)^2 = 25$$

The center of the circle is at $(-3, 1)$ and the radius is $\sqrt{25}$ or 5 units.



30. The center of the ellipse is the midpoint of the major axis.

$$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{4 + (-6)}{2}, \frac{1 + 1}{2}\right)$$
$$= \left(\frac{-2}{2}, \frac{2}{2}\right)$$
$$= (-1, 1)$$

The length of the major axis of an ellipse is $2a$ units. In this ellipse, the length of the major axis is the distance between the points at $(4, 1)$ and $(-6, 1)$. This distance is 10 units.

$2a = 10$
$a = 5$

The length of the minor axis of an ellipse is $2b$ units. In this ellipse, the length of the minor axis is the distance between the points at $(-1, 3)$ and $(-1, -1)$. This distance is 4 units.

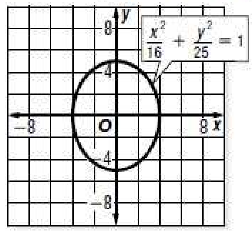
$2b = 4$
$b = 2$

Since the major axis is horizontal, the equation is of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. An equation of the ellipse is $\frac{[x-(-1)^2]}{5^2} + \frac{(y-1)^2}{2^2} = 1$ or $\frac{(x+1)^2}{25} + \frac{(y-1)^2}{4} = 1$.

31. The equation is of the form $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$. The center of the ellipse is at $(0, 0)$. Since $a^2 = 25$, $a = 5$. Since $b^2 = 16$, $b = 4$. The length of the major axis is $2(5)$ or 10 units. The length of the minor axis is $2(4)$ or 8 units. Use the equation $c^2 = a^2 - b^2$ to determine the value of c.

$c^2 = a^2 - b^2$
$c^2 = 25 - 16$
$c^2 = 9$
$c = 3$

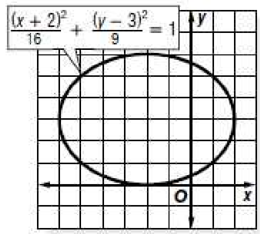
Since $c = 3$, the foci are at $(0, 3)$ and $(0, -3)$.



32. The equation is of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. The center of the ellipse is at $(-2, 3)$. Since $a^2 = 16$, $a = 4$. Since $b^2 = 9$, $b = 3$. The length of the major axis is $2(4)$ or 8 units. The length of the minor axis is $2(3)$ or 6 units. Use the equation $c^2 = a^2 - b^2$ to determine the value of c.

$c^2 = a^2 - b^2$
$c^2 = 16 - 9$
$c^2 = 7$
$c = \sqrt{7}$

Since $c = \sqrt{7}$, the foci are at $(-2 \pm \sqrt{7}, 3)$, or $(-2 + \sqrt{7}, 3)$ and $(-2 - \sqrt{7}, 3)$.



33. $x^2 + 4y^2 - 2x + 16y + 13 = 0$

$(x^2 - 2x + \square) + 4(y^2 + 4y + \square) = -13 + \square + 4(\square)$

$(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = -13 + 1 + 4(4)$

$(x - 1)^2 + 4(y + 2)^2 = 4$

$\frac{(x-1)^2}{4} + \frac{4(y+2)^2}{4} = 1$

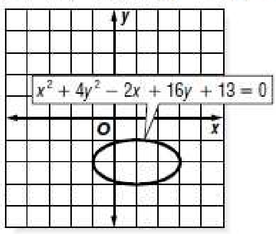
$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{1} = 1$

The equation is of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

The center is at $(1, -2)$. Since $a^2 = 4$, $a = 2$. Since $b^2 = 1$, $b = 1$. The length of the major axis is 2(2) or 4 units. The length of the minor axis is 2(1) or 2 units. Use the equation $c^2 = a^2 - b^2$ to determine the value of c.

$c^2 = a^2 - b^2$
$c^2 = 4 - 1$
$c^2 = 3$
$c = \sqrt{3}$

Since $c = \sqrt{3}$, the foci are at $(1 \pm \sqrt{3}, -2)$, or $(1 + \sqrt{3}, -2)$ and $(1 - \sqrt{3}, -2)$.



34. The transverse axis of the hyperbola is vertical, so the equation is of the form $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$.

The center of the hyperbola is the midpoint of the segment connecting the vertices.

$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$= \left(\frac{2+2}{2}, \frac{5+1}{2}\right)$
$= \left(\frac{4}{2}, \frac{6}{2}\right)$
$= (2, 3)$

The length of the transverse axis is $2a$ units. In this hyperbola, the length of the transverse axis is the distance between the vertices. This distance is 4 units.

$2a = 4$
$a = 2$

The length of the conjugate axis of a hyperbola is $2b$ units. In this hyperbola, the length of the conjugate axis is 6 units.

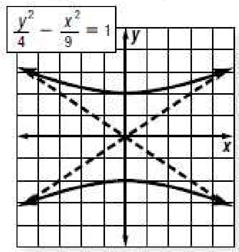
$2b = 6$
$b = 3$

Since $a = 2$, $a^2 = 4$. Since $b = 3$, $b^2 = 9$. Substitute the values for a^2 and b^2. An equation of the hyperbola is $\frac{(y-3)^2}{4} - \frac{(x-2)^2}{9} = 1$.

35. The center of this hyperbola is at the origin. According to the equation, $a^2 = 4$ and $b^2 = 9$, so $a = 2$ and $b = 3$. The coordinates of the vertices are $(0, 2)$ and $(0, -2)$. Use the equation $c^2 = a^2 + b^2$ to determine the value of c.

$c^2 = a^2 + b^2$
$c^2 = 4 + 9$
$c^2 = 13$
$c = \sqrt{13}$

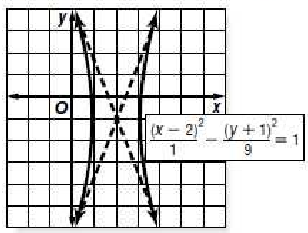
Since $c = \sqrt{13}$, the foci are at $(0, \sqrt{13})$ and $(0, -\sqrt{13})$. The equations of the asymptotes are $y = \pm\frac{a}{b}x$ or $y = \pm\frac{2}{3}x$.



36. The center of this hyperbola is at $(2, -1)$. According to the equation, $a^2 = 1$ and $b^2 = 9$, so $a = 1$ and $b = 3$. The coordinates of the vertices are $(2 \pm 1, -1)$, or $(3, -1)$ and $(1, -1)$. Use the equation $c^2 = a^2 + b^2$ to determine the value of c.

$c^2 = a^2 + b^2$
$c^2 = 1 + 9$
$c^2 = 10$
$c = \sqrt{10}$

Since $c = \sqrt{10}$, the foci are at $(2 \pm \sqrt{10}, -1)$. The equations of the asymptotes are $y - (-1) = \pm\frac{3}{1}(x - 2)$ or $y + 1 = \pm 3(x - 2)$.



37. $9y^2 - 16x^2 = 144$

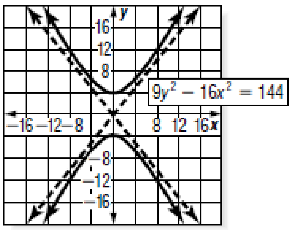
$\frac{9y^2}{144} - \frac{16x^2}{144} = 1$

$\frac{y^2}{16} - \frac{x^2}{9} = 1$

The center of this hyperbola is at the origin. According to the equation, $a^2 = 16$ and $b^2 = 9$, so $a = 4$ and $b = 3$. The coordinates of the vertices are $(0, 4)$ and $(0, -4)$. Use the equation $c^2 = a^2 + b^2$ to determine the value of c.

$c^2 = a^2 + b^2$
$c^2 = 16 + 9$
$c^2 = 25$
$c = 5$

Since $c = 5$, the foci are at $(0, 5)$ and $(0, -5)$. The equations of the asymptotes are $y = \pm\frac{4}{3}x$.



38. $16x^2 - 25y^2 - 64x - 336 = 0$

$16(x^2 - 4x + \square) - 25y^2 = 336 + 16(\square)$

$16(x^2 - 4x + 4) - 25y^2 = 336 + 16(4)$

$16(x - 2)^2 - 25y^2 = 400$

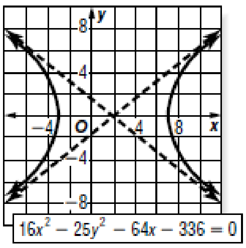
$\frac{16(x-2)^2}{400} - \frac{25y^2}{400} = 1$

$\frac{(x-2)^2}{25} - \frac{y^2}{16} = 1$

The center of this hyperbola is at $(2, 0)$. According to the equation, $a^2 = 25$ and $b^2 = 16$, so $a = 5$ and $b = 4$. The vertices are at $(2 \pm 5, 0)$, or $(-3, 0)$ and $(7, 0)$. Use the equation $c^2 = a^2 + b^2$ to determine the value of c.

$c^2 = a^2 + b^2$
$c^2 = 25 + 16$
$c^2 = 41$
$c = \sqrt{41}$

Since $c = \sqrt{41}$, the foci are at $(2 \pm \sqrt{41}, 0)$. The equations of the asymptotes are $y = \pm\frac{4}{5}(x - 2)$.



39. $x^2 + 4x - y = 0$

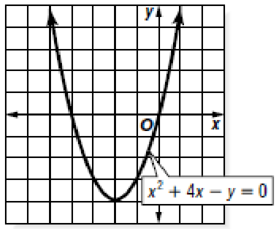
$(x^2 + 4x + \square) - y = \square$

$(x^2 + 4x + 4) - y = 4$

$(x + 2)^2 - y = 4$

$(x + 2)^2 - 4 = y$

The graph of the equation is a parabola.

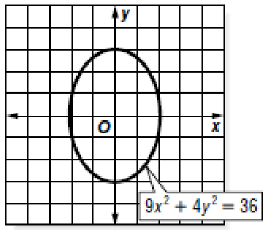


40. $9x^2 + 4y^2 = 36$

$\frac{9x^2}{36} + \frac{4y^2}{36} = 1$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

The graph of the equation is an ellipse.



41. $-4x^2 + y^2 + 8x - 8 = 0$

$y^2 - 4(x^2 - 2x + \square) = 8 - 4(\square)$

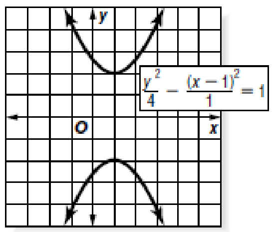
$y^2 - 4(x^2 - 2x + 1) = 8 - 4(1)$

$y^2 - 4(x - 1)^2 = 4$

$\frac{y^2}{4} - \frac{4(x-1)^2}{4} = 1$

$\frac{y^2}{4} - \frac{(x-1)^2}{1} = 1$

The graph of the equation is a hyperbola.



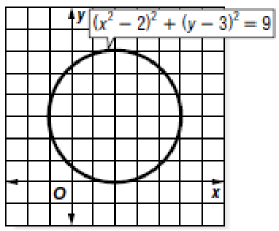
42. $x^2 + y^2 - 4x - 6y + 4 = 0$

$(x^2 - 4x + \square) + (y^2 - 6y + \square) = -4 + \square + \square$

$(x^2 - 4x + 4) + (y^2 - 6y + 9) = -4 + 4 + 9$

$(x - 2)^2 + (y - 3)^2 = 9$

The graph of the equation is a circle.



43. In this equation, $A = 7$ and $C = 9$. Since A and C are both positive and $A \neq C$, the graph is an ellipse.