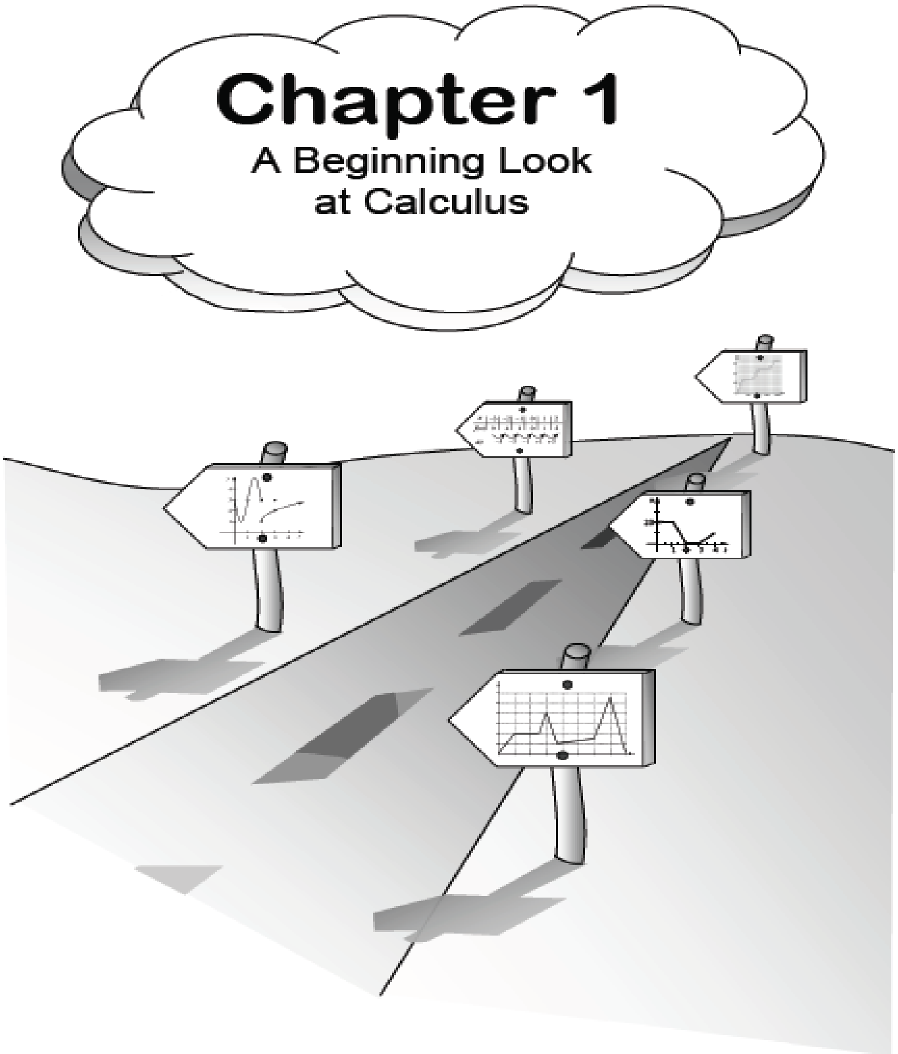

Chapter 1
A Beginning Look
at Calculus

WHAT IS CALCULUS GOOD FOR?

Welcome to calculus! What is calculus, anyway? We will spend this year answering this question. However, in the first activity, you and your team will work on a problem that illustrates the kind of questions calculus can help solve. Many of the mathematical and problem solving skills you already have will enable you to find approximate solutions. As this course progresses, the methods of calculus will enable you to find precise solutions. You will find that you may learn better if you show your work clearly and completely. You can explain your ideas in writing, using equations, graphs, tables, and well-labeled calculations. Correctness, completeness, and clarity all count!

Chapter Goals

Build a positive team climate for learning calculus.

Review pre-calculus topics.

Find volumes of solids geometrically.

Investigate finite differences

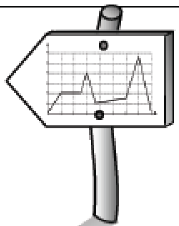
Explore slope.

Examine the relationship between position and velocity graphs.

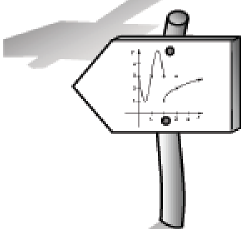
In this chapter, you will:

- Review topics from previous courses such as piecewise functions, compositions, inverses, even & odd functions, domain & range, and horizontal and vertical asymptotes.
- Develop the concepts of slope and slope functions.
- Study how particular functions change by examining finite differences.
- Examine both the velocity and distance graph of an object in motion to find average velocity and acceleration.

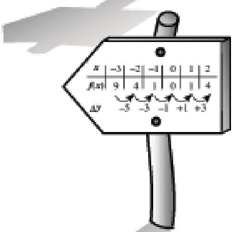
Chapter Outline



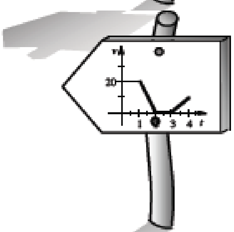
Section 1.1 You will start the year with your team exploring area under the curve and slope at a point, both major themes of calculus.



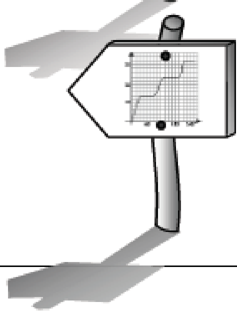
Section 1.2 You will review various types of functions such as piecewise, composite, inverse, and even/odd functions. You will describe their graphs including stating their domain and range. You will explore the behavior of graphs at the far left and right as well as any holes or vertical asymptotes within the function.



Section 1.3 You will study slope through finite differences. You will learn to write slope statements and sketch slope functions.



Section 1.4 You will use graphs to analyze motion. You will find average velocity on distance graphs and velocity graphs. You will discover the relationships between total distance, displacements, velocity, speed and acceleration.



Section 1.5 You will examine area and slope on distance and velocity graphs to find a relationship between them.

Chapter 1 Teacher Guide

Section	Lesson	Days	Lesson Objectives	Materials	Homework
1.1	1.1.1	1	Applying Rates and Distance	Lesson 1.1.1 Res. Pg.	1-2 to 1-10
1.2	1.2.1	1	Piecewise Functions and Continuity	None	1-20 to 1-28
	1.2.2	1	End Behavior and Horizontal Asymptotes	None	1-36 to 1-43
	1.2.3	1	Holes, Vertical Asymptotes, and Approach Statements	None	1-51 to 1-60
	1.2.4	1	Composite Functions and Inververse Functions	None	1-68 to 1-76
	1.2.5	1	Attributes of Even and Odd Functions	None	1-82 to 1-91
	1.2.6	1	Design a Flag (optional)	None	1-93 to 1-95
1.3	1.3.1	1	Finite Differences	None	1-101 to 1-109
	1.3.2	1	Slope Statements and Finite Differences of Non-Polynomials	None	1-113 to 1-121
	1.3.3	1	The Slope Walk	• CBL with Motion Detector or CBR • Distance/Time program • Overhead graphing calculator (or computer display)	1-125 to 1-132
1.4	1.4.1	1	Distance and Velocity	• CBL with Motion Detector or CBR	1-138 to 1-146
	1.4.2	1	Average Velocity on a Position Graph	• Lesson 1.4.2 Res. Pg. • Colored pencils	1-152 to 1-161
	1.4.3	1	Average Velocity on a Velocity Graph	None	1-167 to 1-175
	1.4.4	1	Acceleration	None	1-181 to 1-189
1.5	1.5.1	1	Area and Slope	Lesson 1.5.1 Res. Pg.	1-194 to 1-203
Chapter Closure		Varied Format Options			

Total: AP Calculus 14-15 days plus optional time for Chapter Closure

Objectives:

May 20, 2013

Find the area under a curve
Find the slope of a line at a point

Section 1.1
Exploring the Fundamental Theorem of Calculus

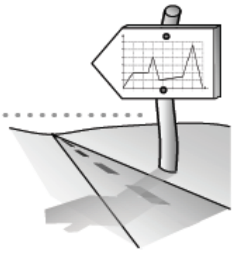
Question/ Summary:
DLIQ... also make a story about the truck driver...

Homework: 1.2 - 1.10
Due: Tuesday, May 28, 2013

Reminders: 11.1 parts 1 and 2; Khan... trig and functions

1.1.1 How are speed and distance related?

Applying Rates and Distance



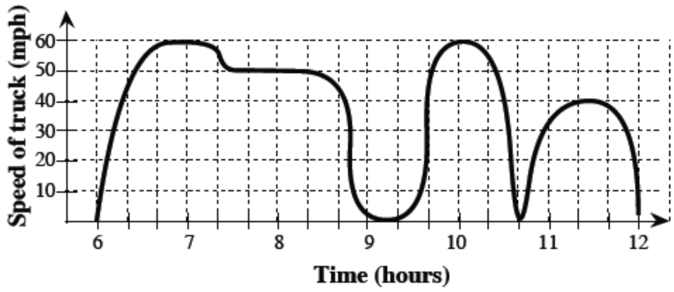
The situations you will encounter in calculus at first seem like problems you have seen before. You already have many mathematical methods to *approximate* the solutions. During this calculus course, you will develop tools to find precise solutions.

As both an overview and introduction, this chapter is a good opportunity to reinforce the qualities of good teamwork: mutual respect, self-reliance, good communication and fair division of labor. The ability to work well as a team will increase the effectiveness of future learning.

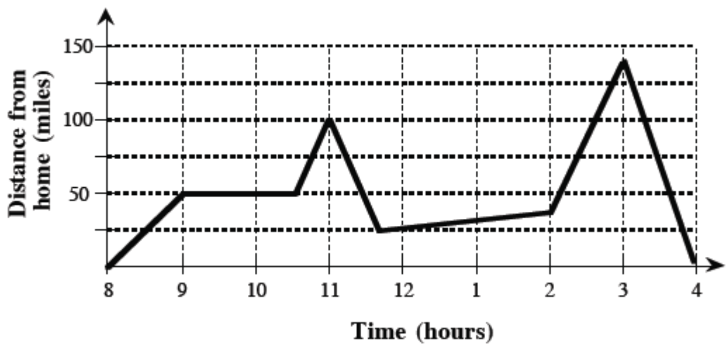
1-1. FREEWAY FATALITIES

Some people think that the number of freeway accidents can be reduced if cars and trucks were prevented from speeding. In fact, in some countries, trucks are legally required to have special devices (called tachographs) on their wheels which records the truck's speed at all times.

a. A graph showing the speed of a truck in miles per hour over a 6-hour period is shown below. Estimate the total distance the truck has traveled during this time. Then explain how you could get more accurate estimates using the same graph.



b. The graph below shows the distance traveled by a *different* truck over an 8-hour time period. Make and justify as many statements as you can about the truck's speed at various times.



c. Look back at your work in both graphs. The answers you got related to the *geometry* of each graph.

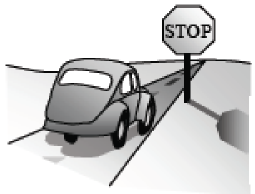
For instance, in the first graph, confirm that the truck traveled about 27 miles from 6:00 a.m. to 6:40 a.m., and 53 miles from 7 a.m. to 8 a.m. What do 27 and 53 represent geometrically in the first graph?

In the second graph, confirm that from 8 a.m. to 9 a.m., the speed of the truck is 50 mph. What does this 50 represent geometrically about the second graph?

Review & Preview

Math Notes

Volumes of Standard Solids



The following formulas are useful for finding the volumes of typical geometric figures. Note the B is the *base area* of the solid.

Prism
$V = Bh$

Cylinder
$V = \pi r^2 h$

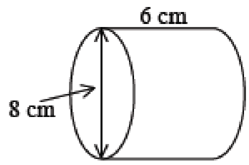
Sphere
$V = \frac{4}{3}\pi r^3$

Pyramid
$V = \frac{1}{3}Bh$

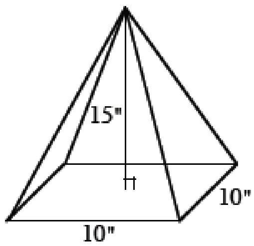
Cone
$V = \frac{1}{3}\pi r^2 h$

1-2. Find the volume of each of the following solids.

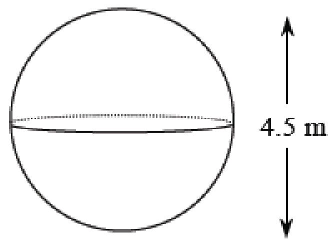
a.



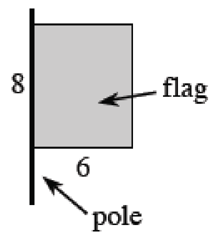
b.



c.



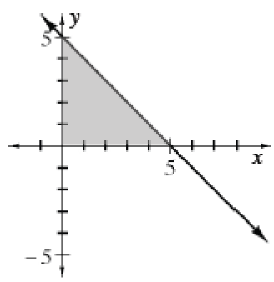
1-3. We will define a “flag” as a geometric area attached to a line segment (its “pole”). An example is shown at right.



a. Imagine rotating the flag about its pole and describe the resulting three-dimensional figure. Draw a picture of this figure on your paper.

b. Find the volume of the rotated flag.

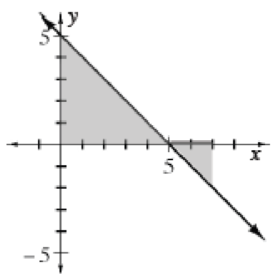
1-4. Examine the graph the function $f(x) = 5 - x$ at right.



a. Find the area of the shaded region using geometry.

We will use the notation $A(f, 0 \le x \le 5)$ to represent “The area between the function and the x-axis” over the interval from $x = 0$ to $x = 5$.

b. Notice that the line dips below the x-axis when $x > 5$. We will consider the area *below* the x-axis as negative. Find $A(f, 0 \le x \le 7)$.



c. Find k if $A(f, 0 \le x \le k) = 0$. Show how you obtained your solution clearly and completely.

1-5. Quickly sketch the function $g(x)=\sqrt{16-x^2}$.

a. State the domain and range of $g(x)$.

b. Use geometry to find $A(g, 0 \le x \le 4)$.

c. Find $A(g, -4 \le x \le 4)$.

d. What is the relationship between the answers of (b) and (c)?

1-6. A car travels 50 miles per hour for 2 hours and 40 miles per hour for 1 hour.

a. Sketch a graph of velocity vs. time. Label the axes with units.

b. Fill out the table below for the distance vs. time.

Time	0.5	1	1.5	2	2.5	3
Distance						

c. Graph the function of distance vs. time. Label the axes with units.

1-7. TRANSLATING FUNCTIONS

a. Graph the function $y=\frac{2}{3}x^2$. On the same set of axes graph a translation of the function that is shifted one unit to the right and five units down. Write the equation of the translated function.

b. Does the same strategy work for $y=\frac{2}{3}x$? Write an equation that will shift $y=\frac{2}{3}x$ one unit to the right and five units down.

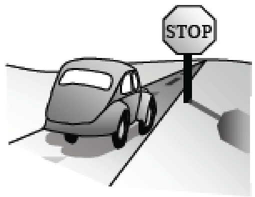
c. Compare the graphs of $y=-\frac{1}{2}x$ and $y=-\frac{1}{2}(x+2)+3$. Describe their similarities and differences.

d. Explain how you know that the graph of $y=-9(x+1)-6$ goes through the point $(-1, -6)$ and has a slope of -9.

e. Sketch the graph of $y=5(x-2)-1$ quickly.

1-8. Find the equation of the line through the point $(-5, -2)$ with a slope of -3 using the method developed in problem 1-7. Refer to the Math Notes box below for help.

MATH NOTES



Point-Slope Form of a Line

When given the slope of a line and one point on the line, we can find the equation of the line using the **point-slope form**. This is the translation of the origin of the line $y = mx$ to the point (h, k). The equation is of the form:

$$y = m(x - h) + k$$

Sometimes, you will see the point-slope form of a line also written as: $y - k = m(x - h)$, which is an equivalent expression.

1-9. Now you know *two* general equations used to write the equation of a line:

$$y = mx + b \quad \text{and} \quad y = m(x - h) + k$$

Under what circumstances is each equation easier to use? For parts (a) through (c) below, determine which method is best with the given information. Then, write the equation for the line.

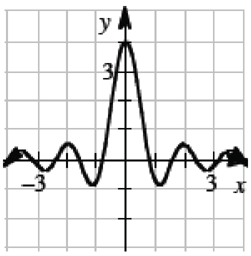
a. $m = -\frac{2}{5}$ and through (–6, 2)

b. $m = 3$ and $b = -6$

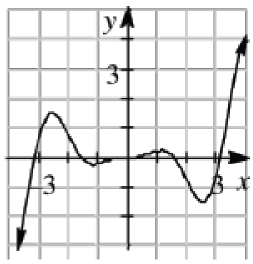
c. Through (2, 8) and (1, 3).

1-10. For each function $f(x)$ sketched below, sketch $f(-x)$ and compare it with $f(x)$. Then describe its symmetry.

a.



b.

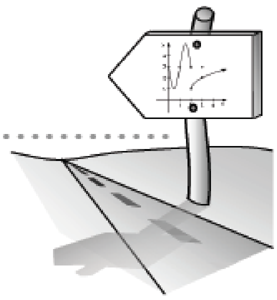


c. EVEN AND ODD FUNCTIONS--INFORMALLY

A function that is symmetric with respect to the y-axis, like that in part (a) above, is called an **even** function. A function that is symmetric with respect to the origin, such as that in part (b), is called an **odd** function.

Sketch examples of even and odd functions. Include how you can test whether a function is even or odd. Then list some famous even/odd functions from your parent graphs, and the symmetries associated with even and odd functions.

1.2.1 What if the function is in pieces?



Piecewise Functions and Continuity

1-11. On the same axis, graph these two functions:
$g(x)=\frac{1}{2}x+1$ for $x<4$, and $h(x)=-x+6$ for $x\geq 4$.

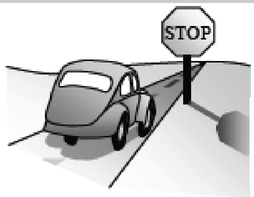
a. What happens to the graph at $x=4$?

b. When we combine parts of several functions to make a single function, we call it a **piecewise function**. Just as the graphs can be drawn on the same set of axes, the algebraic functions of g and h can be "pieced together" as a single function.

We will call this function $f(x)=\begin{cases}\frac{1}{2}x+1, & x<4\\ -x+6, & x\geq 4\end{cases}$.

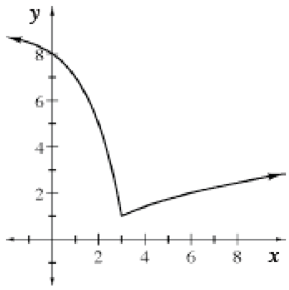
Evaluate: $f(0)$, $f(4)$, and $f(6)$.

MATH NOTES



Intuitive Notion of Continuity

We will formally define continuity later in Chapter 2. For now, we can say that a function is **continuous** if you can draw the graph of the function without lifting your pencil from the paper.



The graph of a continuous function is shown at right.
The equation of this piecewise functions is

$$y=\begin{cases} 9-2^x & \text{for } x \le 3 \\ \sqrt{x-2} & \text{for } x>3 \end{cases}.$$

The graph is continuous at $x = 3$ because $9-2^3=\sqrt{3-2}$.
Though the graph is connected at $x = 3$, notice that the domains of the individual equations do NOT overlap at $x = 3$. This ensures that the piecewise is a function.

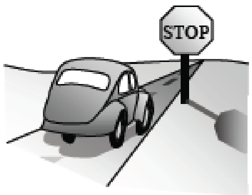
When the y-values of a function are not connected as in problem 1-11, we say that the function is **discontinuous**. The point of discontinuity is the x-value where the function becomes discontinuous. The point of discontinuity in problem 1-11 is $x = 4$.

1-12. Examine the graph of the function $h(x)=\begin{cases} 9-2^x & \text{for } x \le 3 \\ \sqrt{x-2} & \text{for } 3 < x \end{cases}$ in the Math Note above.

a. Is $h(x)$ continuous or discontinuous? Justify your answer.

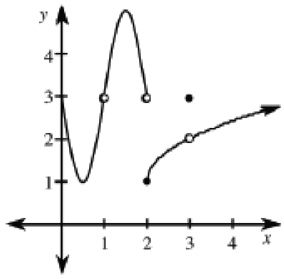
b. Explain why $h(x)$ could also be defined as $h(x)=\begin{cases} 9-2^x & \text{for } x < 3 \\ \sqrt{x-2} & \text{for } 3 \le x \end{cases}$.

MATH NOTES



Domain and Range Notation

There are two accepted forms of notation for domain and range. Examine the graph at right. The domain and range of the function shown can be noted in either **interval** notation or in **set** notation.



Interval Notation: $D=[0,1)\cup(1,\infty)$
$R=[1,\infty)$

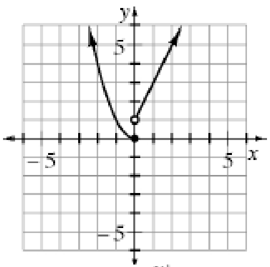
Set Notation: $D=\{x : x\geq 0 \text{ and } x\neq 1\}$ OR $D=\{x : 0\leq x<1 \text{ or } x>1\}$
$R=\{y : y\geq 1\}$ $R=\{y : y\geq 1\}$

1-13. Explain why the two forms of set notation for the domain, D, in the Math Notes box above are equivalent.

1-14. Compare the domains of $f(x)$, $g(x)$, and $h(x)$ below. Explain what aspect of each function limits the domain.

$f(x)=\sqrt{x-25}$ $g(x)=\frac{1}{x-25}$ $h(x)=\log(x-25)$

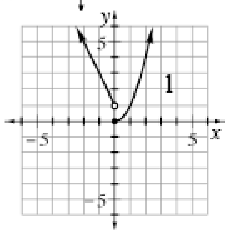
1-15. Given $f(x)=\begin{cases} x^2 & \text{for } x \le 0 \\ 2x+1 & \text{for } x>0 \end{cases}$ and its graph at right, match each transformation to its graph.



A. $f(x+2)=\begin{cases} (x+2)^2 & \text{for } x \le -2 \\ 2(x+2)+1 & \text{for } x>-2 \end{cases}$

B. $f(x)+2=\begin{cases} x^2+2 & \text{for } x \le 0 \\ 2x+3 & \text{for } x>0 \end{cases}$

C. $f(-x)=\begin{cases} (-x)^2 & \text{for } x \ge 0 \\ 2(-x)+1 & \text{for } x<0 \end{cases}$



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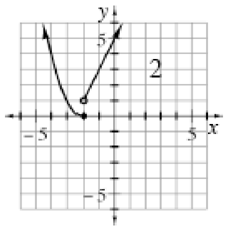

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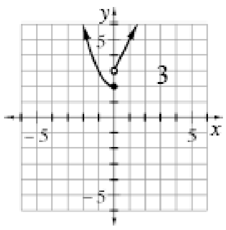


3

a. Find the domain and range for $f(x)$.

b. At what value of x is $f(x)$ is discontinuous?

c. At what value of x is $f(x+2)$ discontinuous? What about $f(x)+2$? Explain the different answers.

1-16. There is a debate among mathematicians over whether $f(x)=|x|$ should be considered a parent graph, or if it is simply a piecewise function such that:

$$f(x)=|x|=\begin{cases}-x & \text{for } x\le 0\\ x & \text{for } x>0\end{cases}.$$

a. Write $g(x)=|x-2|$ as a piecewise function.

b. Write $h(x)=\begin{cases}-(x+5) & \text{for } x\le -5\\ x+5 & \text{for } x>-5\end{cases}$ as an absolute value function.

c. Why do you think this issue is *debatable*?

1-17. Given the function $f(x)=\begin{cases}x^2+2 & \text{for } x<1\\ -x & \text{for } x\ge 1\end{cases}$,

a. Sketch the graph of $f(x)$.

b. Modify one piece of $f(x)$ so that the function is continuous.

1-18. Find values of a and b so that $g(x)$ is continuous.

$$g(x)=\begin{cases}\sqrt{x+3} & \text{for } x<1 \\ a(x-1)^2+b & \text{for } 1\le x<3 \\ -x+2 & \text{for } x\ge 3\end{cases}$$

1-19. Selected values of a continuous function, $f(x)$, are shown in the table below.

x	-4	-2	0	2	4	6
$f(x)$	7	5	3	1	1	3

a. If the graph of $f(x)$ has one unique minimum value, what do you think this value is? Explain your thinking.

b. Could the graph of $f(x)$ be a parabola? If so, find a possible equation of $f(x)$. If not, explain.

c. Could the graph of $f(x)$ be an absolute value function? If so, find a possible equation of $f(x)$. If not, explain.

d. Is it possible that $f(3) = 10$. Explain.

e. Is it possible that $f(x)$ has a vertical asymptote at $x = 3$? Explain.

Review & Preview

1-20. Let $g(x)=\begin{cases} 2 & \text{for } 0 \le x \le 3 \\ 3 & \text{for } 3 < x \le 5 \\ 7 & \text{for } 5 < x \le 8 \end{cases}$.

a. Sketch the graph of $g(x)$. Is this function continuous? Explain.

b. Shade the area between $g(x)$ and the x-axis. Find $A(g, 0 \le x \le 8)$.

c. The equation $g(x)$ is an example of a **step function**. Why do you think it is called a step function?

1-21. Given the following functions, compute the given values.

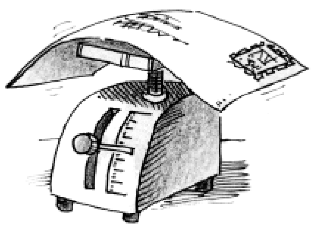
a. $f(x)=\begin{cases} 4-3x & \text{for } x \le 1 \\ x^2 & \text{for } x > 1 \end{cases}$ Find $f(0)$, $f(1)$, and $f(3)$.

b. $f(x)=\begin{cases} \sqrt{x} & \text{for } x < 3 \\ 3-x & \text{for } x \ge 3 \end{cases}$ Find $f(1)$, $f(3)$, and $f(9.4)$.

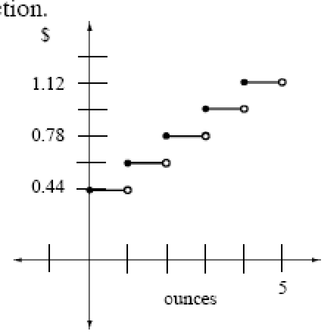
c. $f(x)=\begin{cases} -x & \text{for } x \le 0 \\ \frac{5}{x} & \text{for } 0 < x \le 1 \\ 6-2x & \text{for } x > 1 \end{cases}$ Find $f(-3)$, $f(0)$, $f(0.5)$ and $f(4)$.

d. Sketch a graph of $f(x)$ in part (c) above.

1-22. In order to mail a letter in the United States, postage must be paid based on the weight of the letter. Although rates are tied to the number of ounces, the U. S. Post Office does not allow for payments of partial ounces. For example, if your letter weighs 6.1 ounces, you must pay for 7 ounces.



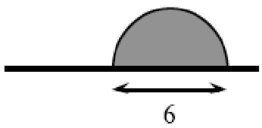
A graph showing the postage rates for letters weighing fewer than five ounces for the year 2009 is shown at right. This is another example of a step function.



a. How much would you pay for a letter weighing 2.9 ounces? For 3 ounces? For 3.1 ounces?

b. Write a piecewise function that determines the postage rates for letters weighing between 0 and 5 ounces. Let x represent ounces, and y represent cost.

1-23. A semi-circular flag is shown attached to a "pole" at right.



a. Imagine rotating the flag about its pole and describe the resulting three-dimensional figure. Draw a picture of this figure on your paper.

b. Find the volume of the rotated flag.

1-24. Sketch the graph of the piecewise function $g(x)$ below.

$$g(x)=\begin{cases}2^x & \text{for } x\le 2\\ 3x-2 & \text{for } x>2\end{cases}$$

a. State the domain and range of $g(x)$.

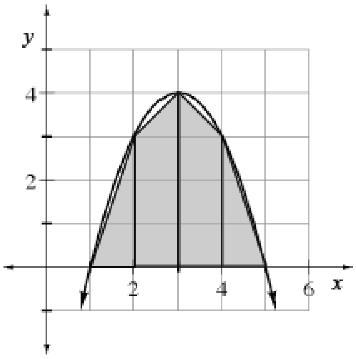
b. Is $g(x)$ continuous at $x=2$? Explain.

c. Is $g(x)$ continuous on all values of x?

1-25. The parabola $y=-(x-3)^2+4$ is graphed at right. Four trapezoids of equal width are inscribed for $1\le x\le 5$.

a. Use the combined area of these trapezoids to approximate the area under the parabola for $1\le x\le 5$.

b. Is this area higher or lower than the true area under the parabola? Explain.



1-26. Find the exact value of each of the following trig expressions.

a. $\sin\frac{5\pi}{3}$ b. $\tan\frac{7\pi}{6}$ c. $\sec\frac{5\pi}{4}$ d. $\csc\pi$

1-27. Sketch the graph of $y=\frac{1}{x}$.

a. Why does this graph have a vertical asymptote? What is the equation of that asymptote?

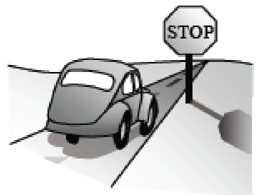
b. State the equation of the horizontal asymptote.

c. Alter $y=\frac{1}{x}$ so that the vertical asymptote is $x=1$ and the horizontal asymptote is $y=3$.

1-28. Use polynomial division to simplify the following rational expressions.

a. $\frac{x^3+2x^2-3x+4}{x+3}$ b. $\frac{x^4-5x^2+3x-3}{x-2}$

MATH NOTES



Polynomial Division with Remainders

The examples below show two methods for dividing polynomials. Both have remainders that we write as fractions.

$$\frac{x^4-6x^3+18x-1}{x-2}$$

Using Long division:

$$
\begin{array}{r}
x^3-4x^2-8x+2 \\
x-2\,\overline{\big)\,x^4-6x^3+0x^2+18x-1} \\
\underline{x^4-2x^3} \qquad\qquad\qquad \\
-4x^3+0x^2 \qquad\qquad \\
\underline{-4x^3+8x^2} \qquad\qquad \\
-8x^2+18x \qquad \\
\underline{-8x^2+16x} \qquad \\
2x-1 \\
\underline{2x-4} \\
3
\end{array}
$$

Remainder

Final Answer: $x^3-4x^2-8x+2+\frac{3}{x-2}$

Using Generic Rectangles:

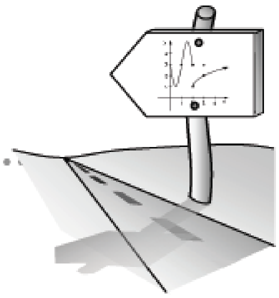
Remainder

	x^3	$-4x^2$	$-8x$	$+2$	
x	x^4	$-4x^3$	$-8x^2$	$+2x$	3
-2	$-2x^3$	$+8x^2$	$+16x$	-4	

$x^4 \quad -6x^3 \quad +0x^2 \quad +18x \quad -1$

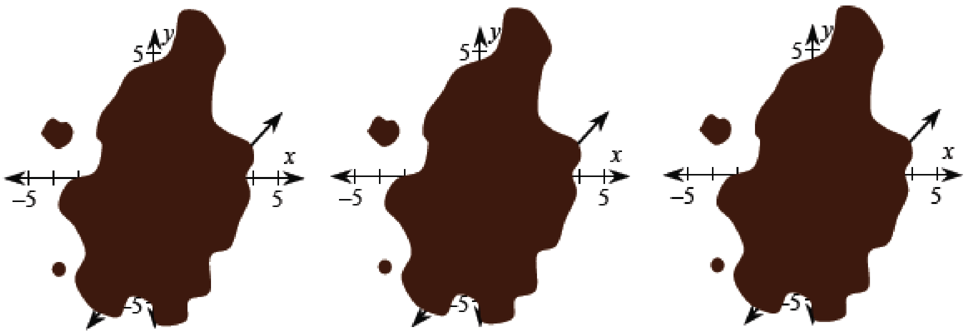
Final Answer: $x^3-4x^2-8x+2+\frac{3}{x-2}$

1.2.2 Did you look far left and far right?



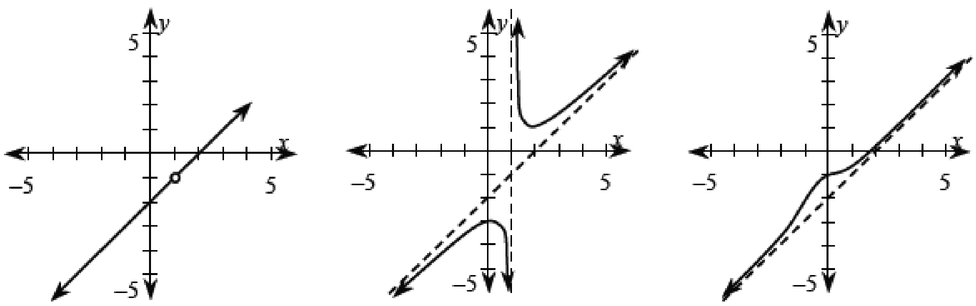
End Behavior and Horizontal Asymptotes

1-29. Eunsun spilled coffee on her math book, making some of the graphs difficult to read!



a. Help Eunsun determine the equation of each function graphed above.

b. When Eunsun arrived at school, she looked at Rudy's book and discovered that the graphs looked like this:



Compare the graphs in part (a) with the graphs in part (b). Use the words vertical asymptote and hole in your explanation.

c. The actual equations are:

$$y=\frac{x^2-3x+2}{x-1} \qquad y=\frac{x^2-3x+3}{x-1} \qquad y=\frac{x^3-2x^2+x-1}{x^2+1}$$

Use algebra to simplify each expression. What do you notice?

1-30. Even though Eunsun's equations were incorrect, to her surprise, she received partial credit on the assignment. The next day, Eunsun's teacher used her homework to teach a new concept: END BEHAVIOR!

Graph $f(x) = -x + 2 + \frac{2x}{x^2+1}$ on your calculator.

a. Use your calculator to "zoom out" so you can look left and right. Find the end behavior function for the graph.

b. Describe the connection between the equation of the end behavior function and the equation of the original function?

c. The linear equation in part (a) is called a slant asymptote. Explain why the name is appropriate.

1-31. Sketch a graph of $y = \frac{1}{x^2+1} + 3$. Then look far to the left and to the right. Determine the end behavior of this graph.

1-32. Estimate the end behavior of the following continuous functions, given selected values shown on the tables below.

a.

x	$f(x)$
–1000	499
–950	470
–900	440
–100	300
–50	200
–25	300
–10	500
0	600
10	500
25	300
50	200
100	300
900	440
950	470
1000	499

b.

x	$f(x)$
–1000	6.99
–950	6.97
–900	6.92
–100	6.42
–50	6.20
–25	4.80
–10	–200
0	DNE
10	–200
25	4.80
50	6.20
100	6.42
900	6.92
950	6.97
1000	6.99

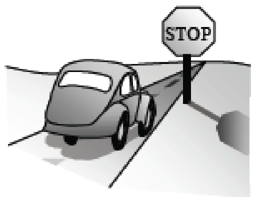
1-33. If $f(x) = \frac{6x^2 - x + 3}{3x+1}$:

a. Demonstrate that $f(x) = 2x - 1 + \frac{4}{3x+1}$.

b. Using your graphing calculator and a suitable window, graph $f(x)$. Then, find the end behavior function for $f(x)$.

c. What is the connection between the end behavior function and $f(x)$ as written in part (a)?

MATH NOTES

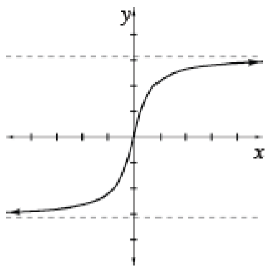


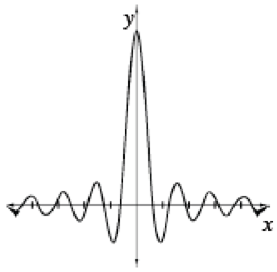
Horizontal Asymptotes
(informal)

Suppose that a function $f(x)$ approaches a horizontal line as x approaches ∞. Then this line is a **horizontal asymptote** to the function. $f(x)$ may also have a horizontal asymptote as x approaches $-\infty$. Note: Slant linear asymptotes can also exist, as shown in problem 1-29.

It is important to note that horizontal and slant asymptotes MAY BE CROSSED.

Later in this course we will give a formal definition of asymptotes using limits.





1-34. Use algebra to find the end behavior of each equation below.

i. $y=\frac{-5x^2+3}{x-2}$ *ii.* $y=\frac{2x+4}{3x-2}$ *iii.* $y=\frac{x^2-4x+4}{x-2}$ *iv.* $y=\frac{2x^3+2x}{x}$

a. What type(s) of asymptotes does each of the functions in part (a) have? Justify your answer.

b. Write a function that has a horizontal asymptote of $y=\frac{5}{4}$.

1-35. Sketch the following functions. If it is impossible to sketch, explain why.

a. $f(x)$ has a horizontal asymptote $y = 2$.

b. $g(x)$ has horizontal asymptotes $y = 2$ and $y = -3$.

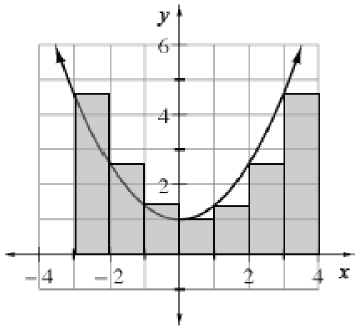
c. $h(x)$ has horizontal asymptotes $y = 2$, $y = -3$, and $y = 7$.

d. $j(x)$ passes through the origin and has a horizontal asymptote at $y = 0$.

e. $k(x)$ has a horizontal asymptote at $y = 2$ and a slant asymptote $y = x$.

Review & Preview

1-36. The graph of $y = \frac{2}{5}x^2 + 1$ is shown at right.



a. Approximate $A(y, -3 \le x \le 3)$ by finding the sum of the areas of the 6 left endpoint rectangles as shown. (The height of a left endpoint rectangle is determined by the function's height at the left x-value.)

b. Is the approximation from part (a) too high or too low? How can you tell?

c. Now, sketch this function with 6 right endpoint rectangles and compute the approximate area.

d. You should have found the same answers using right and left endpoint rectangles. Would this be true for all functions? If so, explain why. If not, explain what was special about the function above that made the area estimates equal. Give an example of a case where the area estimates would be different.

1-37. A car travels at a rate of $20x + 30$ miles per hour for $0 \le x \le t$.

a. Sketch a velocity graph and label the axes with the correct units.

b. Shade the area under the curve for $0 \le x \le t$. What does this area represent?

c. What are the units of the area? Explain how you know.

d. Compute the distance traveled for $0 \le x \le 2$.

1-38. If $f(x) = \frac{3}{x^2} + 1$,

a. Find the domain and range of $f(x)$.

b. Find expressions for $f(-x)$, $f(\sqrt{x})$, and $f(x+h)$.

1-39. Graph the following functions on your graphing calculator and zoom out until you can clearly see its end behavior. Then, find the end behavior function.

a. $y=1-\frac{1}{x}$ b. $y=\frac{3x^2}{6x+1}$

1-40. State the domain for each of the functions below.

a. $f(x)=\frac{x}{x^2+1}$ b. $g(x)=\frac{1}{x}-\frac{x}{x+1}$

c. $h(x)=\sqrt{x^2-9}$ d. $k(x)=\frac{\log(x-3)}{\sqrt{x+4}}$

1-41. Wei Kit knows that roots can be re-written using exponents. Study his examples below:

EXAMPLES: $\sqrt{x}=x^{1/2}$ $\left(\sqrt[5]{z}\right)^2=z^{2/5}$ $\sqrt[3]{m^2}=m^{2/3}$

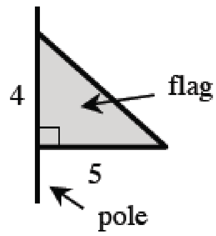
Use Wei Kit's method to rewrite the following radicals using exponents.

a. $\sqrt{k^7}$ b. $\sqrt[3]{t^4}$ c. $\left(\sqrt{n}\right)^4$ d. $\sqrt[5]{b^{31}}$

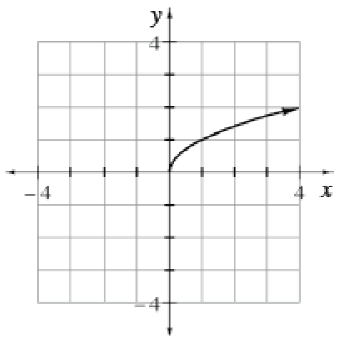
1-42. Imagine rotating the flag at right about its pole.

a. Describe the resulting three-dimensional figure. Draw a picture of this figure on your paper.

b. Find the volume of the rotated flag.



1-43. Copy the graph at right. Then complete it so it will have the symmetry described below.



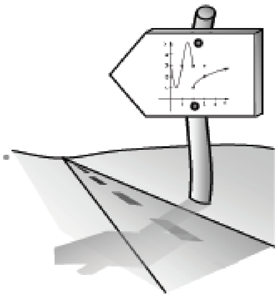
a. Reflection symmetry across the y-axis.

b. Reflection symmetry across the x-axis.

c. Point symmetry at the origin. (This means a 180° rotation around the origin leaves the graph unchanged.)

d. Recall the definitions of even and odd functions. For each drawing, state if it is even, odd or neither.

1.2.3 What happens in the middle?



Holes, Vertical Asymptotes, and Approach Statements

1-44. For $f(x)=\frac{x^2+5x+3}{x+4}$ and $g(x)=\frac{x^2+5x+4}{x+4}$:

a. Draw a careful sketch of each function. Use a dashed line for an asymptote and an open circle for a "hole" (a single point which the graph appears to go through, but where it is actually undefined).

b. For both $f(x)$ and $g(x)$, find the equation of all asymptotes and the coordinates of all missing points (called "holes").

c. Find the domain and range of $f(x)$ and $g(x)$.

1-45. HOLES AND ASYMPTOTES

With your team, write a conjecture that states which rational expressions of the form $\frac{p(x)}{q(x)}$ have a vertical asymptote and which have a "hole." When using your graphing calculator, be sure to use a "friendly window." To get you started, several rational expressions are given below. Be sure to generate your own rational expressions to confirm your conjecture.

Possible rational expressions: $\frac{x^2+2x+1}{x+1}$ $\frac{x^2+2x+2}{x+1}$

$\frac{x^2-5x+6}{x-2}$ $\frac{x^2-5x+6}{x-1}$

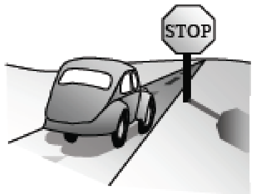
1-46. MORE ON RATIONAL EXPRESSIONS

a. Does $\frac{x^2+5x+6}{x+3} = x+2$? Why or why not? Do the two expressions have the same graph?

b. The expressions $\frac{x^2-5x+6}{x-2}$ and $x-3$ are not quite equal. Add a statement to $\frac{x^2-5x+6}{x-2} = x-3$ to make it true.

c. In previous courses, you may have ignored the domain issue when simplifying algebraic fractions, but from now on, domain considerations will be important. Simplify $\frac{x^2-4}{x-2}$.

MATH NOTES



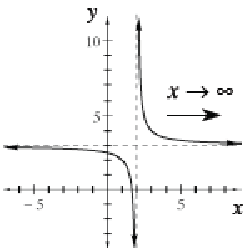
Approach Statements

A statement that describes the behavior of a function at various locations is called an **approach statement**. Although you can use shorthand symbols, such as "$\rightarrow$," you need to write a complete set of statements involving all important approaches of the function.

A *complete* set of approach statements includes the extremes of the domain, as well as any holes or asymptotes. Below is a complete set of approach statements for $y = \frac{1}{x-2} + 3$.

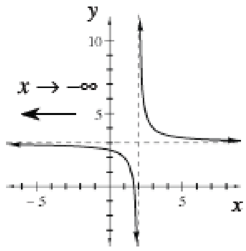
As x approaches infinity, y approaches 3.

As $x \rightarrow \infty$, $y \rightarrow 3$.



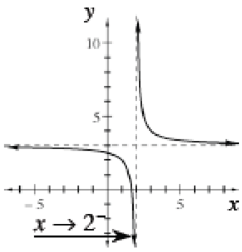
As x approaches negative infinity, y approaches 3.

As $x \rightarrow -\infty$, $y \rightarrow 3$.



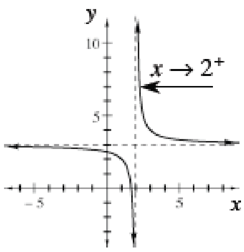
As x approaches 2 from the left, y approaches negative infinity.

As $x \rightarrow 2^-$, $y \rightarrow -\infty$.

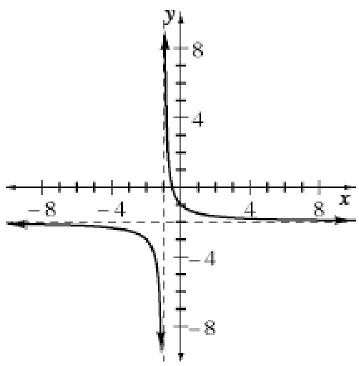


As x approaches 2 from the right, y approaches positive infinity.

As $x \rightarrow 2^+$, $y \rightarrow \infty$.



1-47. Examine the graph of $y=\frac{1}{x+1}-2$ at right. Use the graph to answer the questions below.



a. What does y approach as $x \to \infty$?

b. What does y approach as $x \to -\infty$?

c. What does y approach as $x \to -1^-$ (the symbol "-1^-" means approaching from the negative direction, or from the left)?

d. What does y approach as $x \to -1^+$ (from the positive direction, or from the right)?

e. Name all horizontal and vertical asymptotes.

1-48. Sketch a graph of an even function that has a vertical asymptote at $x = 2$, a hole at $x = -4$ and as $x \to \infty$, $y \to 3$.

1-49. In problem 1-46, the numerator and denominator were both polynomials. When this is not the case, factoring is no longer useful. For each fraction, evaluate $f(0)$. Then,

i. If the function is defined at $x = 0$, state the value at $x = 0$.

ii. If the function is not defined at $x = 0$, use your calculator to sketch a graph. Clearly indicate whether the function has a hole or an asymptote at $x = 0$.

a. $f(x) = \frac{\sin x}{x}$ b. $f(x) = \frac{\sin^2 x}{x}$ c. $f(x) = \frac{\sin x}{x^2}$

d. $f(x) = \frac{\cos x}{x}$ e. $f(x) = \frac{1-\cos x}{x-1}$ f. $f(x) = \frac{1-\cos x}{x}$

1-50. For the following functions, when 2 is substituted for x, the fraction has the **undefined** *(or indeterminate)* **form** $\frac{0}{0}$. State whether the following graphs have holes, asymptotes, or neither at $x = 2$. Explain your answer.

a. $f(x) = \frac{x-2}{x-2}$ b. $g(x) = \frac{(x-2)^2}{x-2}$ c. $h(x) = \frac{x-2}{(x-2)^2}$

d. Sketch and write a function that looks like $y = x - 2$ with a hole at $x = 4$.

1-51. Analyze the graph of $y=\frac{(x+2)(x-1)}{x-1}$ at right.

a. What does y approach as $x \to \infty$?

b. What does y approach as $x \to -\infty$?

c. What does y approach as $x \to 1^-$ (from the left)?

d. What does y approach as $x \to 1^+$ (from the right)?

1-52. Find a function with the following complete set of approach statements. Hint: Start by sketching the graph.

As $x \to 3^+$, $y \to \infty$,
$x \to 3^-$, $y \to -\infty$,
$x \to -\infty$, $y \to 1$, and as
$x \to \infty$, $y \to 1$.

1-53. Convert the following domain and range from interval to set notation. Then sketch a possible function.

$D = (-\infty, 2) \cup (2, \infty)$ $\qquad$ $R = (-\infty, -1) \cup (-1, \infty)$

1-54. On graph paper, sketch the function $g(x)=\sqrt{36-x^2}$. Shade $A(g, 3 \le x \le 6)$, the region between $g(x)$ and the x-axis, for $3 \le x \le 6$.

a. Use geometry to find this area. Hint: Draw in a radius to create two easier regions whose difference is the shaded region.

b. Find $A(g, 0 \le x \le 3)$.

c. Find $A(g, -3 \le x \le 6)$.

1-55. A marathon runner runs a 26.2-mile race. Her distance traveled in miles at time t hours is $p(t) = 7t$.

a. How long did it take her to finish the race?

b. What was her average velocity? Explain your reasoning.

c. Suppose she runs at a constant pace of 7 miles/hour. How far will she have gone in 2 hours?

d. Show how the units cancel using rate $\cdot$ time = distance.

1-56. Wei Kit loves shortcuts! When calculating with fractional exponents, he looks for a way to avoid using his calculator. For example, he found out that $8^{2/3} = 4$ by using the method below:

$$8^{2/3} = (\sqrt[3]{8})^2 = (2)^2 = 4$$

Use Wei Kit's method to evaluate the following expressions:

a. $100^{3/2}$ b. $27^{4/3}$ c. $16^{3/4}$ d. $9^{4/2}$

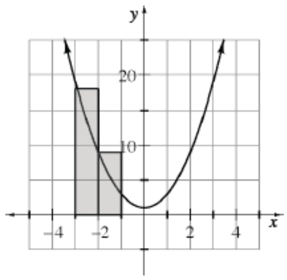
1-57. Sketch a graph of $y = 1 - x^3$. Then complete the following approach statements.

a. As $x \to \infty$, y approaches?

b. As $x \to -\infty$, y approaches?

c. As $x \to 0^-$ (from the left), y approaches?

1-58. Estimate $A(f(x), -3 \le x \le 3)$ for $f(x) = 2x^2 + 1$.



a. Using left endpoint rectangles. The first two rectangles are shown.

b. Using right endpoint rectangles.

c. Using trapezoids. What do you notice? Does this always happen?

1-59. Each of the continuous functions in the table below is increasing, but each increases differently. Match each graph at right with the function that grows in a similar fashion in the table.

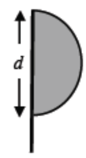
x	1	2	3	4	5	6	7	8	9
$f(x)$	64	68.8	74.6	81.5	89.8	99.7	111.7	126	143.2
$g(x)$	38	52	66	80	94	108	122	136	150
$h(x)$	22	42.9	57.3	68.5	77.6	85.3	92	97.9	103.1

a.

b.

c.

1-60. When the flag to the right is rotated it has a volume of $\left|\frac{243}{2}\pi\right|$ un^3.

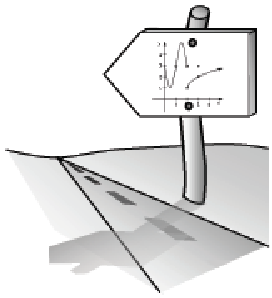


a. Describe the resultant three-dimensional figure.

b. What is the value of d?

c. If the diagram was rotated 90° and the flag was then rotated about a horizontal pole, would the volume change?

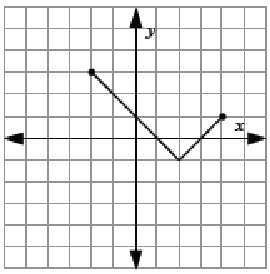
1.2.4 What is a composite function?



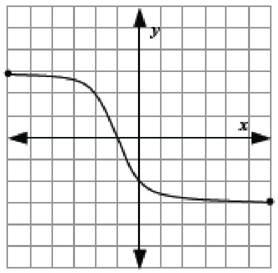
Composite Functions and Inverse Functions

1-61. Given the two functions $f(x)$ and $g(x)$ graphed below:

$f(x)$



$g(x)$



a. State domain and range of $f(x)$.

b. State domain and range of $g(x)$.

c. Find $f(g(-2))$.

d. Find $g(f(-2))$.

e. Find $f(f(3))$.

f. Why can you not evaluate $f(g(5))$?

1-62. If $f(x) = x^2$, $g(x) = x + 1$, and $h(x) = \frac{1}{x}$, express $k(x)$ as compositions of $f(x)$, $g(x)$, and $h(x)$.

For example, $(x+1)^2$ can be expressed as $f(g(x))$.

a. $k(x) = \frac{1}{x^2}$

b. $k(x) = \frac{1}{x} + 1$

c. $k(x) = x^4$

d. $k(x) = \frac{1}{x^2+1}$

1-63. Given $f(x) = 2^x$ and $g(x) = \sqrt{1-x}$, answer the questions below. Use interval notation.

a. Find the domain and range of $f(x)$ and $g(x)$.

b. Find $f(g(x))$ and state its domain.

c. Find $g(f(x))$ and state its domain.

1-64. INVERSE FUNCTIONS

Let $h(x) = 3x + 2$ and $j(x) = \frac{x-2}{3}$.

a. Find $h(j(x))$. What do you notice?

b. Functions such that $f(g(x)) = g(f(x)) = x$ are called **inverse functions**. Explain why this notation would show that f and g are inverse functions.

c. Find a function g such that $f(g(x)) = x$ and $f(x) = e^x + 2$.

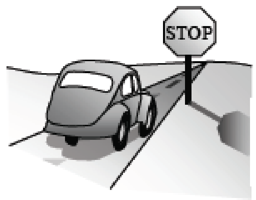
1-65. An inverse function undoes what a function does. For example, $\sin\frac{\pi}{6}=\frac{1}{2}$, which means the sine function takes the angle $\frac{\pi}{6}$ and returns the ratio $\frac{1}{2}$. Therefore the *inverse sine* function takes the ratio $\frac{1}{2}$ and returns the angle $\frac{\pi}{6}$. The notation for inverse functions can be confusing; the inverse of f is written f^{-1}. The inverse sine function is written $\sin^{-1}(x)$. $\sin^{-1}(x)$ is also referred to as $\arcsin(x)$.
Note: $\sin^{-1}(x)\neq\frac{1}{\sin(x)}$!

Write each of these statements entirely in symbols.

a. The inverse sine of $\frac{1}{2}$ is $\frac{\pi}{6}$.

b. When the inverse of the function g is applied to 7, the result is 5.

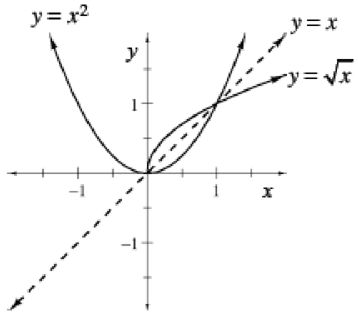
MATH NOTES



Inverse Functions

We say that $f(x)$ and $g(x)$ are **inverse functions** if $f(g(x)) = x$ for all x in the domain of g and $g(f(x)) = x$ for all x in the domain of f. We write $f^{-1}(x)$ for the inverse of $f(x)$. So $g = f^{-1}$ and $f = g^{-1}$.

If we graph $f(x)$ and $f^{-1}(x)$ on the some set of axes then their graphs are symmetric across the line $y = x$. Note: We must restrict the domain of some functions in order for the inverse to be a function.



Some important pairs of inverse functions are $h(x) = x^2$ for $x \geq 0$ and $h^{-1}(x) = \sqrt{x}$ for $x \geq 0$; $j(x) = \sin x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and $j^{-1}(x) = \sin^{-1}(x)$ for $-1 \leq x \leq 1$.

If a function $f(x)$ satisfies the horizontal line test, then f^{-1} exists.

1-66. Solve for x.

a. $f(x) = 2^x$

b. $g(x) = \frac{x+1}{x}$

c. Now find the inverses of $f(x)$ and $g(x)$. What do you notice?

1-67. a. Study the table for the functions $f(x)$ and $g(x)$ at right. $f(x)$ does not have an inverse function. Explain why not.

x	$f(x)$	$g(x)$
–2	5	–3
–1	8	–1
0	9	0
1	8	2
2	5	3

b. Evaluate.

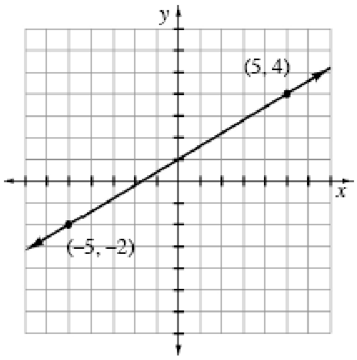
i. $g^{-1}(2)$ *ii.* $f(g^{-1}(2))$ *iii.* $g^{-1}(g(-2))$

c. If $h(3)=4$ and $j(x)=h^{-1}(x)$, find $j(4)$.

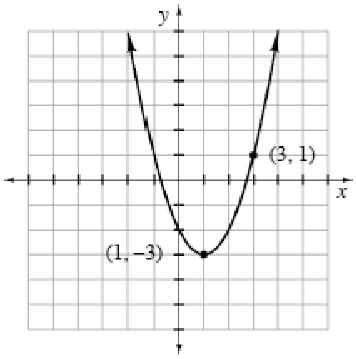
Review & Preview

1-68. Find a possible function for each of the following graphs. Verify your equation on your graphing calculator.

a.



b.



1-69. Much of this course will focus on *change*. Examine two ways a line changes:

a. Sketch $f(x) = 2x + 3$. Find $f(0)$, $f(1)$, $f(2)$, and $f(3)$. How are the function values changing as x increases?

b. Sketch $f(x) = -3x + 10$. Find $f(0)$, $f(1)$, $f(2)$, and $f(3)$. How are the function values changing as x increases?

1-70. Selected values of a continuous *even* function are shown below.

x	0	1	2	3
$f(x)$	0	2	4	6

a. Find $f(-1)$, $f(-2)$ and $f(-3)$.

b. Sketch a possible graph of $f(x)$ on the domain $-3 \le x \le 3$.

c. Sketch another possible graph of $f(x)$ on the domain $-3 \le x \le 3$.

d. Could the graph of $f(x)$ be a parabolic function? If so, find a possible equation of $f(x)$. If not, explain.

1-71. Find the domain of each of the following functions.

a. $f(x) = \sqrt{x+2}$

b. $f(x) = \frac{1}{x-4} + 3$

c. $f(x) = \log(x - 4)$

d. $f(x) = \sqrt{\frac{2-x}{x}}$

1-72. Sketch $f(x) = 3\sqrt{x+1}$ on $0 \le x \le 6$ three times, on three different sets of axes.

a. Review your work from problems 1-25 and 1-36. Use a similar process to approximate $A(f, 0 \le x \le 6)$ using:

i. Six left endpoint rectangles.

ii. Six right endpoint rectangles.

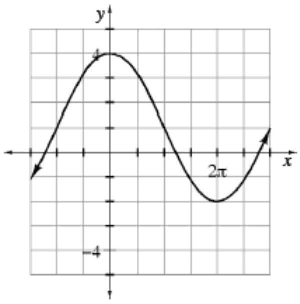
iii. Six trapezoids.

b. Which approximations were over (greater than) estimates of the actual area? Which were under estimates? Explain.

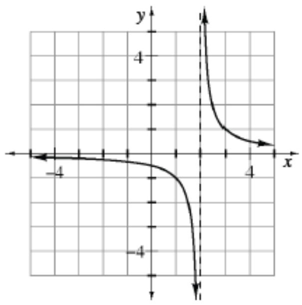
c. Which approximation is more accurate? Explain.

1-73. Use interval notation to state the domain and range of each function below.

a.



b.



1-74. A can of soda is 42°F when purchased. Over the course of the next few hours, the temperature of the soda slowly rises. During an experiment, Shibisha used a thermometer and recorded the temperature at various times, t, shown in the table below.

time(min)	0	10	30	45	60	75	80
temp (°F)	42	51	58	63	66	68	69

a. Sketch a graph of this temperature over time.

b. When is the temperature changing the fastest? How does the graph tell you this?

c. Approximately how fast is the temperature changing during the first 10 minutes? How can you tell?

1-75. Helen thinks $\sqrt{x^2} = x$. Felicia does not agree.

a. Use various values of x to check whether or not Helen is correct.

b. Find an accurate expression for $\sqrt{x^2}$.

1-76. Each of the continuous functions in the table below is increasing, but each increases differently. Match each graph at right with the function that grows in a similar fashion in the table.

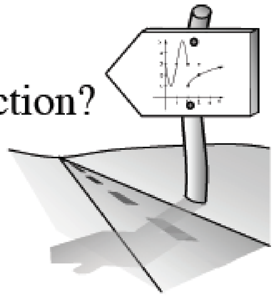
x	1	2	3	4	5	6	7	8	9
$f(x)$	15.5	19.0	22.5	26.0	29.5	33.0	36.5	40.0	43.5
$g(x)$	1	2	4	8	16	32	64	128	256
$h(x)$	12	76	108	124	132	136	138	139	139.5

a.

b.

c.

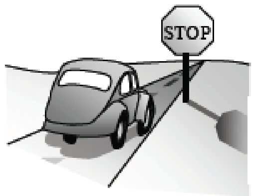
1.2.5 What is the symmetry of an odd function?



Attributes of Even and Odd Functions

1-77. On two sets of axes, sketch $y = \sin x$ on $[-2\pi, 2\pi]$ and $y = \cos x$ on $[-2\pi, 2\pi]$. Describe their symmetries.

MATH NOTES



Even and Odd Functions

A function $f(x)$ is called an **even function** if, for all x in its domain, $f(-x) = f(x)$.

A function $f(x)$ is called an **odd function** if, for all x in its domain, $f(-x) = -f(x)$.

Example: If $f(x) = 2x^3 + \sin(x)$, then $f(-x) = 2(-x)^3 + \sin(-x)$

$$= 2(-1)^3 x^3 + (-1)\sin(x)$$
$$= -2x^3 - \sin(x) = -(2x^3 + \sin(x))$$

Therefore $f(-x) = -f(x)$, so f is odd.

1-78. Each of the following functions is either even, odd, or neither. Use the formal definitions to prove it. Then describe the type of symmetry that each function has.

a. $f(x) = x^2$

b. $g(x) = 2x^3$

c. $h(x) = 2 + x^4$

d. $j(x) = 2 + x^5$

e. $k(x) = \sin 2x$

f. $j(x) = \arctan x$

1-79. Robert wonders about even and odd functions a lot. He wants to know what happens when you combine even functions with odd functions. If $f(x)$ is even and $g(x)$ is odd, find out if the following combinations of $f(x)$ and $g(x)$ are even, odd, or neither. Be sure to explain your analysis.

a. $f(x)+g(x)$

b. $f(x)\cdot g(x)$

c. $f(g(x))$

d. $g(f(x))$

e. $|f(x)|$

f. $|g(x)|$

1-80. Let $f(x)$ be an even function and $g(x)$ be an odd function, both with domain $\{-3, -2, \ldots, 2, 3\}$ and $h(x) = g(f(x))$.

a. On your paper, complete the table of values below.

x	$f(x)$	$g(x)$	$h(x)$
-3	1	1	
-2	2	2	
-1	1	1	
0	0	0	
1			
2			
3			

b. What can you conclude about $h(x)$?

c. Find the following values. If it is impossible, justify why.

i. $h(4)$ *ii.* $g^{-1}(2)$ *iii.* $f(g(-1))$

1-81. If f is an odd function such that $y = 2$ is a horizontal asymptote, which of the following must be true?

i. $y = -2$ is a horizontal asymptote.

ii. $x = 2$ is a vertical asymptote.

iii. $x = -2$ is a vertical asymptote.

Review & Preview

1-82. Are the following functions even, odd, or neither?

a. $y = x^{1/3}$ b. $y = -x^2 + 4$ c. $y = x^3 + x^2 + 1$

1-83. If $f(x) = x^2 + 5x$ and $g(x) = x + 3$, evaluate each of these expressions.

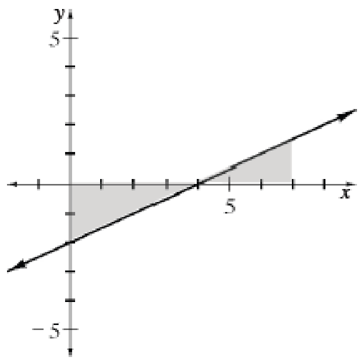
a. $f(-2)$ b. $g(-2)$ c. $f(g(-2))$

d. $g(f(-2))$ e. $f(f(-2))$ f. $g(g(-2))$

1-84. Examine the graph of the function $f(x) = 0.5x - 2$ at right.

a. Find the area of the shaded region using geometry. (Recall that the area below the x-axis is considered negative.)

b. Find k if $A(f, 0 \le x \le k) = 10$. How did you find your solution?



1-85. Graph $f(x) = 2x + \frac{1}{x}$. Find $b(x)$, its end-behavior function.

1-86. A flag is defined by the region between the x-axis and:

$$f(x) = \begin{cases} -x + 5 & \text{for } 1 \le x \le 3 \\ 2 & \text{for } 3 < x \le 6 \end{cases}.$$

Find the volume generated when the flag is rotated about the x-axis.

1-87. Review the directions for writing approach statements in the Math Notes box before problem 1-47. Then write a complete set of approach statements for $y = 3^{-x} + 1$.

1-88. If $y = \frac{x}{x+1}$, approximate $A(y, 0 \le x \le 4)$ using 8 left endpoint rectangles of equal width.

1-89. If $f(x) = x^2 + 5$ and $g(x) = x + 3$, find and simplify the expressions below.

a. $f(g(x))$

b. $g(f(x))$

c. $f^{-1}(-6)$

d. $g^{-1}(-6)$

1-90. Simplify the following functions. *Without a calculator*, sketch each graph, showing roots, holes and asymptotes. Then, state the domain in parts (a) and (b) using **interval** notation and the domain in parts (c) and (d) using **set** notation.

a. $y = \frac{x^2-4}{x^3+3x^2-10x}$

b. $y = \frac{9-x^2}{2x+6}$

c. $y = \frac{x^2-9x-18}{x^2+3x-18}$

d. $y = \frac{x^2-6x+9}{15-5x}$

1-91. The domain of the function f is the set of x-values such that $x > 0$. The range of f is the set of y-values such that $-2 < y \le 5$.

a. Sketch a possible graph for f.

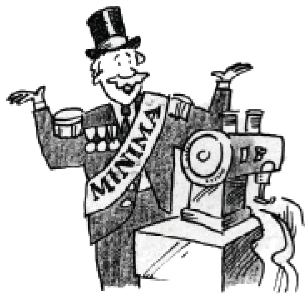
b. Find the domain and range of $f(x-2)+1$.

c. Find the domain and range of $f^{-1}(x)$.

1.2.6 What piecewise functions can you use?

Design a Flag

1-92. DESIGN A FLAG



A tiny country, Minima, is finally about to receive recognition from the United Nations! However, before the ceremony is held to make Minima an official republic, a special flag needs to be designed.

The new president has bestowed upon you the responsibility of designing this flag. He does not want just a colorful flag - he also wants a unique shape designed. He also wants a flag that when rotated will have a volume of 18π cubic units, as there are 18 states that make up the union.

a. Design a horizontal flag that when rotated will have a volume of 18π cubic units. The more interesting and complicated the design, the more appreciative the president will be!

b. Using scaled axes, graph your "flag" and write a piecewise function for its shape. These will be the instructions to the flag seamstress.

c. Explain how you chose your design. What problems did you encounter?

Review & Preview

1-93. Sketch the graph of a function with a domain of $\{x: -2 \le x < 6\}$ and range of $\{y: y > 2\}$. Then sketch a second possible answer.

1-94. Given $f(x) = (x-3)^2 + 4$ and $g(x) = \frac{1}{x-4} - 2$, find and simplify the following:

a. $h(x) = f(x+2) - 5$

b. $k(x) = g(3-x) + 2$

c. Using set notation, find the domain and range for $g(x)$ and $k(x)$.

1-95. Use what you know about functions to quickly sketch the following. Then, check your answer on your graphing calculator.

a. $y = (x-2)^2 + 1$

b. $y = \frac{1}{2}(x-1) - 5$

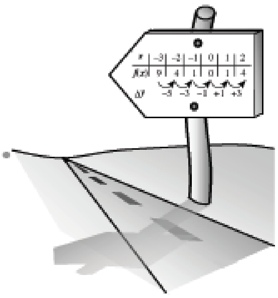
c. $y = -|x-4| - 6$

d. $y = 2^{x+1} + 3$

e. Find the domain and range of the function in part (d). Then find the domain and range of its inverse.

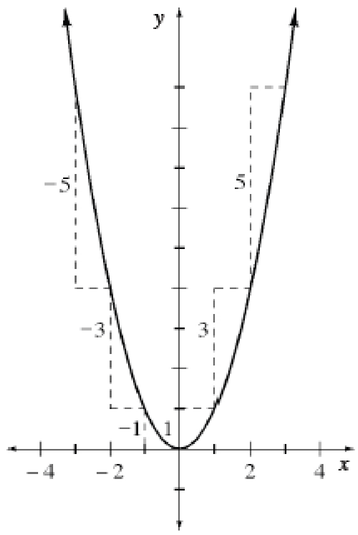
f. Which of the functions above have inverses that are *not* functions? Explain how you know.

1.3.1 How does it change?



Finite Differences

1-96. HOW DOES IT CHANGE?



A large focus in calculus is on how functions change. Whether a function is increasing or decreasing, we measure how that change is occurring. First consider the graph of $f(x)=x^2$. Investigate the graph and note how the function is changing. Slope triangles are shown for the values $-3 \le x \le 3$.

The table below are the **finite differences**, Δy, which are the differences between consecutive y-values.

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

Δy $\quad -5 \quad -3 \quad -1 \quad +1 \quad +3 \quad +5$

Patterns among the finite differences reveal interesting information about the way a function in changing. According to the finite differences on the table above, how are the y-values changing as x increases? Do you see a pattern?

1-97. Does this pattern hold true for other parabolas? Using a table with finite differences labeled, describe how Δy changes as x increases for the functions listed below. Look for patterns within a table as well as between these different parabolas. Record your findings.

a. $f(x) = 2x^2 - 3x + 1$

b. $f(x) = -3x^2 + 6$

1-98. Based on the results you found above, how do parabolas change? Summarize your findings by using the general equation: $f(x) = ax^2 + bx + c$. In other words, when the graph of a function is a parabola, what will the graph of its finite differences look like?

1-99. Now consider the functions below. Describe how each of the graphs change. Be sure to try a variety of examples to verify your observations.

Constant Functions	Linear Functions	Cubic Functions
$f(x)=a$	$f(x)=ax+b$	$f(x)=ax^3+bx^2+cx+d$

1-100. Make a prediction on the how the graph of $f(x) = x^n$ changes.

Review & Preview

1-101. Rewrite $f(x)=|x|$ as a piecewise function using two linear equations. Describe how the graph changes.

1-102. Find the domain of the following functions.

a. $f(x)=\frac{x-2}{x^2+4}$

b. $g(x)=\frac{\sqrt{x+2}}{x^2+x}$

1-103. **Multiple Choice:** The values of x for which the graphs of $y=x+3$ and $y^2=6x$ intersect are:

a. -3 and 3

b. -3

c. 3

d. 0

e. None of these

1-104. Find the *exact* value(s) of x in the domain $\{x:0\le x<2\pi\}$ if:

a. $\sin x=-\frac{1}{2}$, $\tan x>0$.

b. $\cot x$ is undefined, $\cos x>0$.

c. $\csc x=\sqrt{2}$, $\sin x>\cos x$.

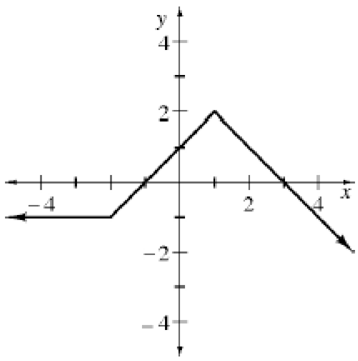
1-105. Given $f(x)=2x^2-3$,

a. Evaluate $f(2)$.

b. Without finding the equation of the inverse, find $f^{-1}(5)$. Explain your process.

c. Solve for x if $f(x+2)-f(x-2)=64$.

1-106. Using the graph of $f(x)$ at right, sketch the following transformations:



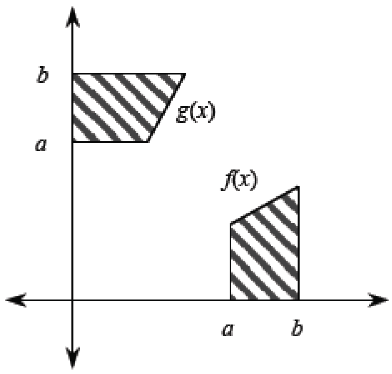
a. $-f(x)$

b. $f(x+3)$

c. $f(x)-2$

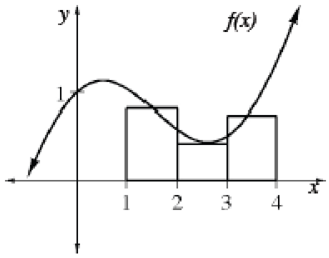
d. $|f(x)|$

1-107. Sandra was playing around with inverses and thinks she has discovered something interesting. She thinks that if $f(x)=g^{-1}(x)$, then the area of the regions shaded at right are equal. Use $f(x)=\frac{1}{3}x+1$ with $a=3$ and $b=5$ to verify Sandra's conjecture.



1-108. To estimate the area under a curve, rectangles are often the easiest shape to use. However, there are different ways to choose the height of the rectangles. You have used left endpoint and right endpoint rectangles. Another way is to use the **midpoint rectangle**, which has a height defined at the midpoint of the interval. For example, for the function

$$f(x)=\frac{1}{2}x+\cos x$$



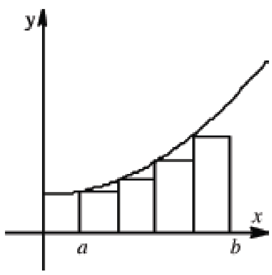
graphed at right, the first rectangle has a height of $f(1.5)\approx 0.821$. Find the height of the other two rectangles then use them to approximate the area under the curve for $1\le x\le 4$.

1-109. WHICH IS BETTER? Part One

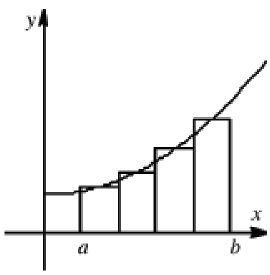
Below are different sets of rectangles to approximate the *same* area under a curve for $f(x)$. Look at the three different sets of rectangles and decide which will best approximate $A(f, a\le x\le b)$ for this function.

a. Explain why your choice will determine the best approximation for the area.

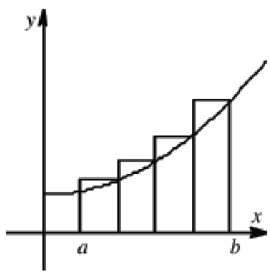
b. Will left endpoint rectangles always be an underestimate for any function? Explain.



Left Endpoint Rectangles

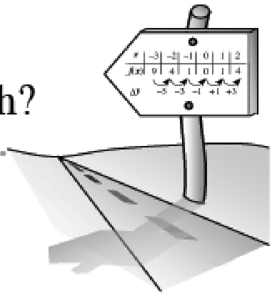


Midpoint Rectangles



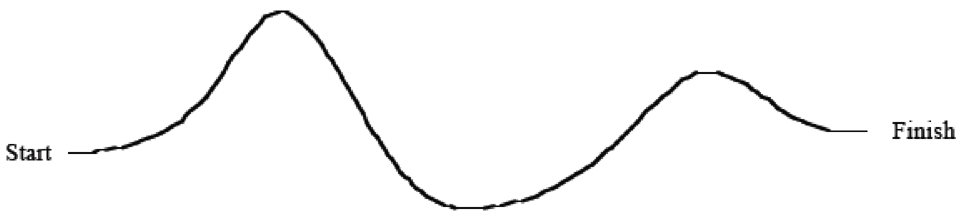
Right Endpoint Rectangles

1.3.2 How can I describe a changing graph?



Slope Statements and Finite Differences of Non-Polynomials

1-110. The path of the roller coaster is shown below.



a. Describe the path so that someone who has not seen it can draw it. Be sure to include words that will help to describe the steepness of the curve as well as its direction.

b. When writing slope statements, it is reasonable to start at the left of the graph and move right---just as you would read a sentence in English. Make a list of words that are useful when describing the path of a graph.

1-111. The following two slope statements describe the *same* function. Read both statements. Then sketch a graph of the function described.

"The graph starts off flat at the left and starts to increase at $x = -3$ until the function flattens out at $x = 0$. Then the value of y decreases until the function flattens out around $x = 2$ and continues flat."

"The graph starts off flat at the left but slowly gets steeper. The slope starts getting really steep at $x = -2$, but at $x = -1$, the slope starts to get less steep. At $x = 0$, the slope is flat for an instant and then gets steeper but negative. At $x = 1$, the slope starts to become less steep again, eventually getting closer and closer to zero slope."

1-112. Finite differences can be used to analyze the slope of a graph at various x-values. Some graphs have predictable slope patterns. For example, in Lesson 1.3.1, you found consistent patterns in the way polynomial functions change. For example, cubic functions change with a quadratic pattern, quadratic functions change with a linear pattern, and linear functions change with a constant pattern. What about other functions?

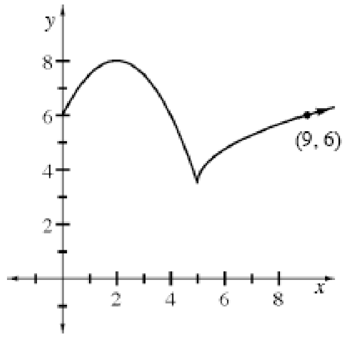
Your team will be assigned one of the function groups listed below to investigate. For the two equations in your function group, complete the following takes:

- Graph the function.
- State the domain and range using appropriate notation.
- Analyze the finite differences.
- Write a slope statement.

Function Group	Equation (a)	Equation (b)
Rational	$f(x)=\frac{1}{x}$	$f(x)=\frac{1}{x^2}$
Trigonometric	$f(x)=\sin x$	$f(x)=\cos x$
Exponential	$f(x)=(0.5)^x$	$f(x)=2^x$
Logarithmic	$f(x)=\log x$	$f(x)=\log_2 x$
Radical	$f(x)=\sqrt{x}$	$f(x)=\sqrt[3]{x}$

Review & Preview

1-113. Create a piecewise defined function that will generate the graph to the right.



1-114. State the domain for each of the following functions.

a. $f(x)=\sqrt{25-x^2}$

b. $g(x)=\log(x+5)$

c. $h(x)=\frac{5x}{x^2-x-12}$

d. $k(x)=\frac{\sqrt{x+2}}{x^2-4}$

1-115. Simplify: $\left(\left(\frac{x^{-1}+x^2}{x}\right)-x+x^{-2}\right)^{-2}$

1-116. Calculus problems often require using one or more of the trigonometric identities to solve problems. Solve each of the following equations where $x\in[0,2\pi]$. Use exact values.

a. $\tan x\cdot\csc x=2$

b. $\sin x\cdot\cos x=\frac{1}{4}$

c. $2\sin^2 x-\cos x-1=0$

d. $\tan x+\cot x=-2$

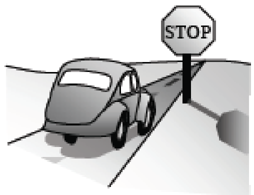
1-117. For each part below, give an example of a function with specified attributes. Provide a sketch of each function.

a. A function with a hole at $x=3$ and an asymptote at $x=-1$.

b. A function with asymptotes at the y-axis and $x=5$ and a hole at $x=-4$.

c. A function with an end behavior function $g(x)=3x-1$.

MATH NOTES



Common Trigonometric Identities

Reciprocal

$\sec x = \frac{1}{\cos x}$ $\csc x = \frac{1}{\sin x}$

$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Angle Sum

$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

Double Angle

$\sin 2a = 2 \sin a \cos a$

$$\cos 2a = \begin{cases} \cos^2 a - \sin^2 a \\ 2\cos^2 a - 1 \\ 1 - 2\sin^2 a \end{cases}$$

1-118. Some of your parent graphs have special qualities that we have studied in this chapter.

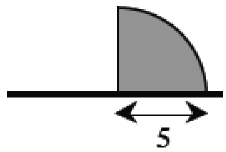
a. Sketch $y = \sin x$ on your paper. Darken in the largest portion of the graph containing $x = 0$ for which the function passes both the horizontal and vertical line tests. State the restricted domain and range for this portion of the graph.

b. We use the darkened portion of the graph to sketch $\sin^{-1} x$, making sure it is a function. Then state the domain and range.

c. Repeat parts (a) and (b) for $y = \cos x$.

1-119. The function $g(x)$ is even. What can you conclude about the inverse of $g(x)$? Explain.

1-120. A flag in the shape of a quarter-circle is shown at right.

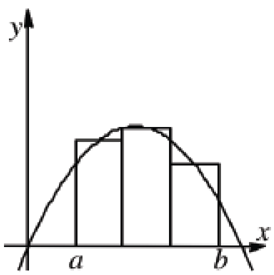


a. Imagine rotating the flag about its pole and describe the resulting three-dimensional figure. Draw a picture of this figure on your paper.

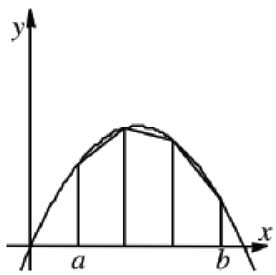
b. Find the volume of the rotated flag.

1-121. WHICH IS BETTER? Part Two

Below is a comparison between using rectangles and trapezoids to approximate the *same* area under a curve for $f(x)$. Decide which method you think will best approximate $A(f, a \le x \le b)$. Then approximate each area if $f(x) = -0.25x(x-9)$, $a = 2$, and $b = 8$ using 3 sections. Compare your results with the actual area $A = 25.5$ square units.

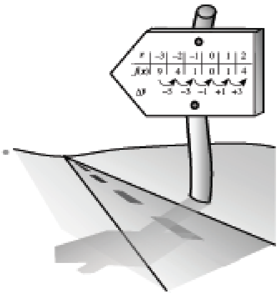


Midpoint Rectangles



Trapezoids

1.3.3 How can I walk a distance graph?



The Slope Walk

1-122. MATCH-A-GRAPH

As your class is attempting to create walks that are provided by your teacher, think about these questions:

- What information does each graph represent?
- What information is required to match the graph precisely?
- What directions would you need to give a classmate so that he or she could match the graph?

1-123. THE SLOPE WALK, Part One

Derive a method for walking each of the parent graphs listed below. In each case the graph will be distance versus time. Your goal is to get the basic shape of each equation, not to go through specific points. Feel free to analyze them in any order.

For each function below:

1. Sketch the graph of $f(x-5)+5$.
2. Walk the sketch. Set the motion detector to record ten seconds worth of data.
3. Write a description of the walk including information about where to start, where to turn around and when you should "speed up" or "slow down".

$f(x)=x^2$	$f(x)=x^3$	$f(x)=2^x$
$f(x)=\lvert x\rvert$	$f(x)=\frac{1}{x}$	$f(x)=\sin x$
$f(x)=x$	$(x-5)^2+(y-5)^2=1$	$f(x)=\sqrt{x}$

1-124. a. Describe how walking the graphs of $y = x - 5$, $y = x + 5$, and $y = -x + 5$ would be similar and different.

b. Christian walked $y = x^2$ and C.J. walked $y = x^3$. How were they the same? How were they different?

c. Adelyn walked $y = 2^x$ and Ara walked $y = \sqrt{x}$. Who had the greatest speed at the beginning? Who had the greatest speed at the end?

Review & Preview

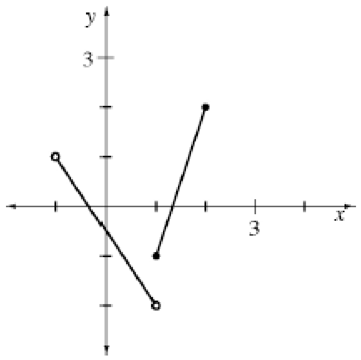
1-125. Carefully graph the function $f(x)=\begin{cases} 3x+4 & \text{for } x<1 \\ -\frac{1}{2}x+5.5 & \text{for } x\ge 1 \end{cases}$.

a. Iveta wants to find $A(f(x), -1\le x\le 5)$, so she decided to split the region into 10 trapezoids and approximate the area. Explain to Iveta why this is not the most efficient method.

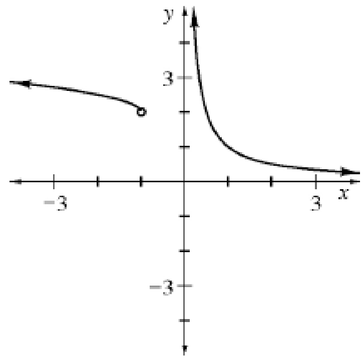
b. Calculate $A(f(x), -1\le x\le 5)$.

1-126. Identify the domain and range of the functions below. Then write a possible piecewise function for each.

a.



b.



1-127. Approximate $A(f, -2\le x\le 3)$ for $f(x)=x^2-x-6$ using 10 left endpoint rectangles.

1-128. While studying the finite differences of a particular function, Neo noticed that the differences changed linearly. What can you tell him about the original function? Also, how do his finite differences change?

1-129. Given: $g(x)=\frac{1}{x^2-x}$

a. Find the domain of $g(x)$.

b. Solve for x if $g(x)=0.5$.

c. Explain why $g(x)$ does not have an inverse function.

1-130. Let $f(x)=x^2$ and $g(x)=\sqrt{x}$. Evaluate the following expressions.

a. $f(3)$

b. $f(-3)$

c. $g(9)$

d. $g(f(3))$

e. $g(f(6))$

f. $g(f(x))$

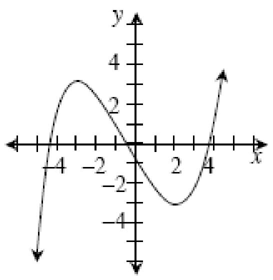
1-131. Examine two ways a line changes:

a. Sketch $f(x)=4x+1$. Find $f(0)$, $f(1)$, $f(2)$, and $f(3)$. How are the function values changing as x increases?

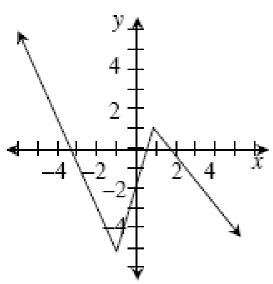
b. Find $A(f, 0 \le x \le a)$ for $a = 0, 1, 2$, and 3. How are the areas changing as a increases?

1-132. For each graph below, state the intervals where the function is increasing and decreasing.

a.

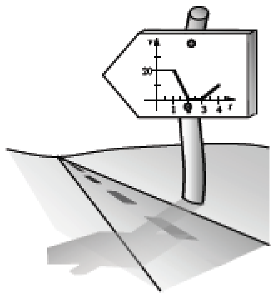


b.



c. For part (a) on the interval in which the function is decreasing, is the rate of decrease constant? How do you know?

1.4.1 Can you walk the walk?



Distance and Velocity

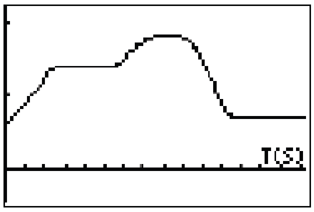
1-133. Shant and Kier each took a 10 second walk and wrote descriptions of their experiences. Complete the tasks below in any order:

- Sketch a graph of each scenario.
- Walk the walk.
- Compare the two walks. Is there a relationship between the two functions?

Shant's Walk: *Start close to the motion detector. Begin walking at a very fast rate and gradually slow down to a stop. Then increase your speed gradually until you are walking as quickly as when you started.*

Kier's Walk: *Start close to the motion detector. Begin walking at a very slow rate and gradually speed up to the fastest speed you can achieve. Then gradually slow down to your starting speed.*

1-134. THE SLOPE WALK, Part Two



During her 15 second Slope Walk, Diamonique walked in front of a motion detector and the graph at right appeared.

a. If the graph represents her distance traveled, describe her motion.

i. State the time interval when Diamonique was walking toward the motion detector? When did she walk away from the motion detector? When did she change directions? Justify your answers.

ii. On what interval of time was she standing still? Justify your answer.

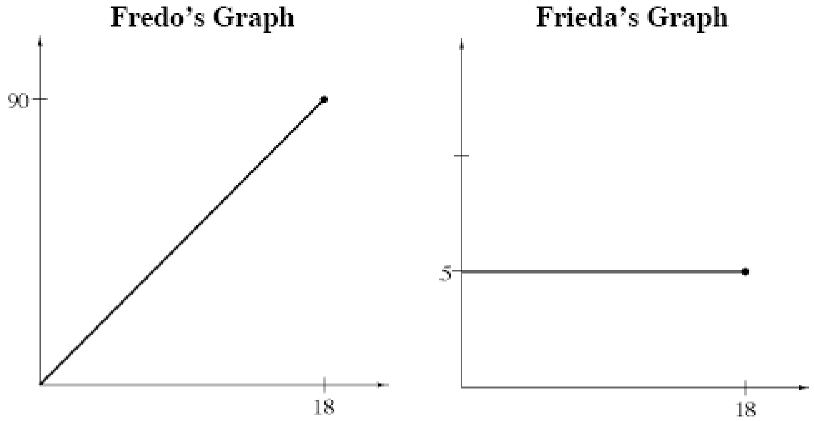
iii. Estimate a time when she was walking fastest. Justify your answer.

iv. Estimate a time when she was slowing down. Justify your answer.

b. Now assume that the graph represents *velocity vs. time*—in other words, let the y-axis represent velocity? Similarly describe her motion.

1-135. FUNDAMENTALLY THE SAME

Fredo (short for Alfredo) and Frieda were both given the responsibility to collect data for a foot race. After watching the event, the students gave the coach graphs of their data, shown below.

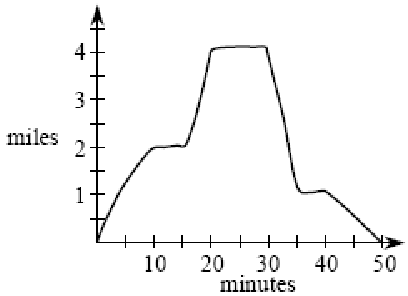


The coach is dismayed because the graphs are different. However, each student is convinced that they watched the same race and that their data is confirmed in the other's graph. Your goal is to help the students convince their coach that the graphs represent the same race.

a. To help the coach understand the graphs, label each with the appropriate (and reasonable) units.

b. Explain how Frieda's graph confirms Fredo's *and* how Fredo's graph confirms Frieda's.

1-136. Ellen rode her bike one morning to visit some friends. Her distance from home is documented in the graph below.



a. Estimate her rate (velocity) in miles per hour for times $t = 7$, 32, and 45 minutes. Describe your strategy.

b. How is Ellen's velocity related to the graph of her distance from home?

c. What was her total distance traveled?

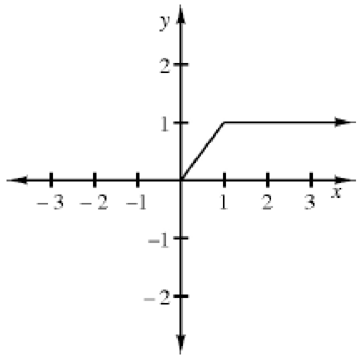
1-137. As a team, summarize what you learned about the relationship between distance and velocity from problem 1-135. Be prepared to share your statements with the class. Problem 1-135 demonstrated a powerful calculus concept: that distance information can come from a velocity graph and velocity information can come from a distance graph. Therefore, at least for simplistic problems, we can determine one from the other using only the basic geometric tools of slope and area.

For the remainder of this chapter, each direction will be examined closely so that some new properties of motion emerge. Central to this work will be the meaning of the terms position/distance and velocity. Define these terms in your own words. Give examples to help clarify your meaning.

1-138. Given a portion of the graph of $f(x)$ at right:

a. Sketch the rest of $f(x)$ if $f(x)$ is even and find $A(f, -3 \le x \le 3)$.

b. Sketch the rest of $f(x)$ if $f(x)$ is odd and find $A(f, -3 \le x \le 3)$.



1-139. Find values of a that that make $h(x)$ continuous.

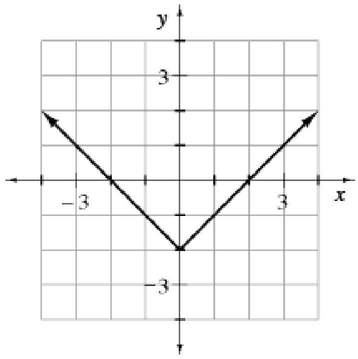
$$h(x) = \begin{cases} \sqrt{x+2} - 1 & \text{for } x < 2 \\ a(x+1)^2 & \text{for } x \ge 2 \end{cases}$$

1-140. Given the function $f(x)$ sketched at right,

a. Sketch $f(-x)$.

b. Sketch $-f(x)$.

c. Sketch $f(f(x))$.



1-141. Find the domain for each of the following functions. Note: The functions mentioned in parts (c) and (d) refer to those in parts (a) and (b).

a. $f(x) = \frac{1}{x+2}$ b. $g(x) = \sqrt{x-4}$

c. $h(x) = f(g(x))$ d. $k(x) = g(f(x))$

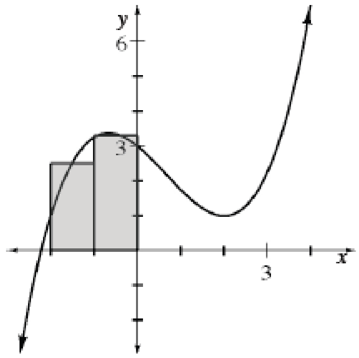
1-142. For each function, use algebra to identify all holes and asymptotes.

a. $f(x) = \frac{x-3}{x^2+4x-21}$ b. $g(x) = \frac{x^{4/3}}{x^2-2x}$

1-143. Velocity is only one example of a rate of change. Name at least two other familiar rates that you encounter in your daily life.

1-144. Let $f(x)$ be a function whose finite differences grow by 4 each time. What kind of function can $f(x)$ be? Give two examples.

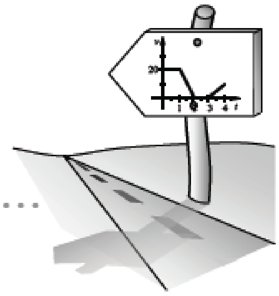
1-145. Cynthia began to draw midpoint rectangles to approximate $A(f, -2 \le x \le 4)$ for $f(x) = \frac{1}{4}x^3 - \frac{1}{2}x^2 - x + 3$. Trace Cynthia's graph and finish drawing the remaining four midpoint rectangles. Then, compute the estimated area.



1-146. Each of these functions has one or more holes and/or asymptotes. Graph them on your graphing calculator and write a complete set of approach statements for each function.

a. $f(x) = \frac{2^x}{x}$

b. $f(x) = \frac{2^x - 1}{x}$



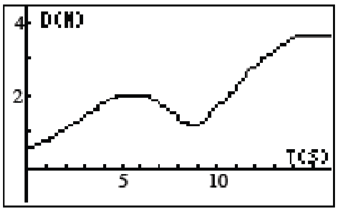
1.4.2 Where is average velocity on a position graph?

Average Velocity on a Position Graph

1-147. WEARY VERONICA, Part One

While exhausted, Veronica produced the distance graph below when walking the Slope Walk. Afterwards, her study team bombarded her with the questions below, to which she tiredly replied, "It's shown here in the graph." The motion detector was set to measure distance in meters and time in seconds.

Examine the graph carefully to find the answers for her teammate's questions.



a. Answer the following questions and justify your answers.

i. "How much time did it take?"

ii. "How far did you travel overall?"

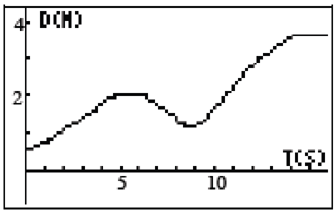
iii. "How far from your starting place did you end up?"

iv. "Did you ever stop? If so, when?"

v. "Did you only walk in one direction?"

b. Explain why the answers to parts (*ii*) and (iii) of the previous problem were not the same.

1-148. WEARY VERONICA, Part Two



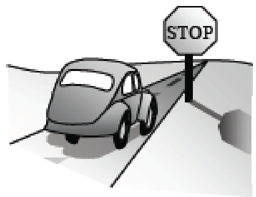
While looking at the graph, Veronica's teammate pointed out that she could have saved her energy and walked from her starting place directly to her ending place instead.

a. On the resource page provided by your teacher, find Veronica's graph. Using a different color, draw what the motion detector would have shown if she had walked directly from her starting position to her ending position at a constant rate taking the same amount of time.

b. What would Veronica's velocity have been had she taken this direct route? This rate is referred to as her **average velocity**.

c. Explain to an Algebra I student the relationship between the graph of Veronica's direct route and her average velocity?

MATH NOTES



Initial Postion, Final Position, Displacement, and Total Distance

Suppose that an object moves between times t_1 and t_2 (usually $t_1 = 0$). The **initial position** is the object's location at time t_1.

The object's **final position** is its location at time t_2.

The **displacement** of the object is the distance between the final position and the initial position.

The **total distance** and object travels is the total of all its motions both forward and back.

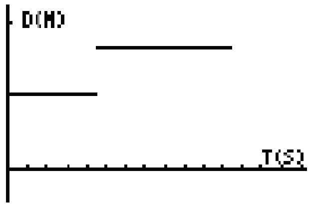
For example, suppose a bug begins at $x = 3$ on the number line, crawls forward 2 units and the backward 4 units. Then its initial position is 3, its final position is 1, its displacement is –2 (because it ended up at a lower number than where it started), and its total distance traveled is 6.

1-149. With your study team (or with the whole class), create a new Slope Walk graph with an average velocity of –2 ft/sec. The graph may not be linear. Copy a graph of the data on the blank axes of the resource page. Use a contrasting color to draw a direct path (secant line) from the starting position to the ending position.

a. With the new graph, answer the questions asked by Veronica's team members in problem 1-147.

b. "Walk" and sketch another non-linear graph that has an average velocity of 0 feet per second. Once again, answer the questions in problem 1-147.

1-150. Poor Agnalia! Her motion detector produced the graph below. "My calculator is broken," she cried jumping up and down, "This graph is physically impossible!" "No it's not," said Amanda, "It's just a piecewise function." Who is correct, Agnalia or Amanda?



1-151. The table below represents the position of a bug crawling along the x-axis of your centimeter grid paper. Time is given in seconds.

time	0	2	5	6	8	10	11	15	16
position	(0, 0)	(2, 0)	(4, 0)	(6, 0)	(9, 0)	(8, 0)	(4, 0)	(4, 0)	(8, 0)

a. Describe the bug's motion. Does it always crawl in the same direction? Is its velocity constant?

b. Compute the bug's average velocity over the following intervals. Use correct units.

i. $0 < t < 16$ *ii.* $0 < t < 2$ *iii.* $8 < t < 11$ *iv.* $5 < t < 15$

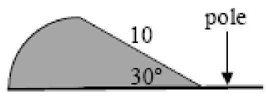
c. Over $0 < t < 15$, will there be a time that the bug is at (5, 0)? Explain.

d. Over $0 < t < 16$, will there be a time at which the bug's average velocity is the same as its actual velocity? Explain.

Review & Preview

1-152. The shaded region at right represents a quarter circle combined with a right triangle "flag."

a. Imagine rotating this flag about its "pole" and describe the resulting three-dimensional figure. Draw a picture of this figure on your paper.



b. Find the volume of the rotated flag.

1-153. For $f(x) = 3x\cos x$, approximate $A(f(x), 0 \le x \le \frac{\pi}{2})$ using two different methods. If the area is approximately 1.712 square units, which of your methods was most accurate? Analyze why that particular method was more accurate.

1-154. If $\sin x = \frac{1}{2}$ and if $0 \le x \le \frac{\pi}{2}$, then *without a calculator* evaluate:

a. $\cos x$ b. $\tan x$

c. $\sec x$ d. $\csc x$

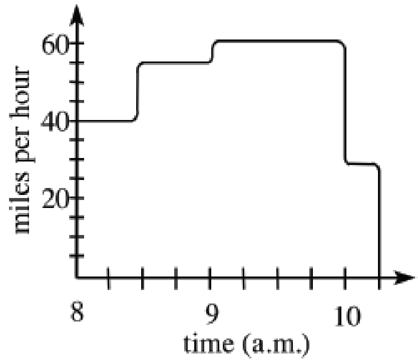
1-155. After Theo used the motion detector, he used his distance-time graph to determine the following properties of his motion. However, he has lost a copy of his graph. Help him re-create a possible graph of his motion.

DETAILS:

- His average velocity was 0.5 feet per second.
- He turned around twice.
- He started while standing 3 feet from the motion detector and began to walk away from it at $t = 0$.
- He walked a total of 9 feet during the 10-second interval.

1-156. DO YOU KNOW THE WAY TO SAN JOSE?

Salima and Karim were driving from Sacramento to San Jose. Salima kept track of the rate as Karim drove. Below is a graph of their rate during the trip.



a. What is the driving distance between Sacramento and San Jose?

b. What was Karim's average speed?

1-157. Find the equation of the line parallel to $9y - 4x = 12$ through the point (6, –7). Write the equation in point-slope form shown in the Math Notes box following problem 1-8.

1-158. Without using a calculator, find the values of each of the following trig expressions.

a. $\sin\frac{5\pi}{6}$

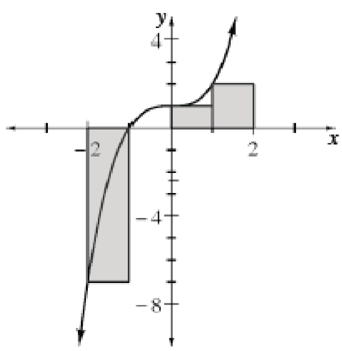
b. $\cos\frac{-3\pi}{4}$

c. $\tan\frac{\pi}{3}$

d. $\sec\frac{5\pi}{3}$

1-159. The height of a right circular cone is twice the radius. If the height of the cone is h, find the volume of the cone using only h.

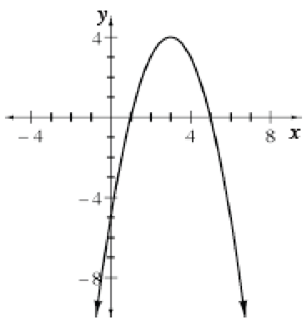
1-160. The function $y = x^3 + 1$ is graphed at right, along with four left endpoint rectangles which approximate the area from $x = -2$ to $x = 2$.



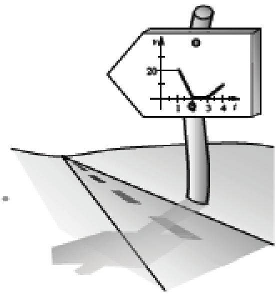
a. Why does it look like there are only three rectangles?

b. Recall that area under the x-axis is negative, while that above the x-axis is positive. Approximate $A(y, -2 \le x \le 2)$ using these four rectangles.

1-161. The parabola $y = -(x-3)^2 + 4$ is graphed at right. Use four trapezoids of equal width to approximate the area under the parabola for $1 \le x \le 5$. Is this area an over or an under estimate of the true area under the parabola?



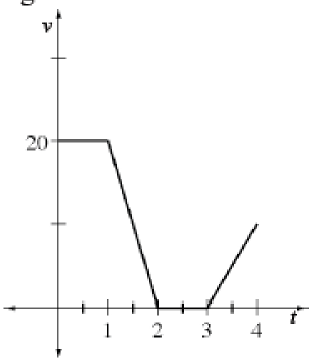
1.4.3 How do I find average velocity on a velocity graph?



Average Velocity on a Velocity Graph

1-162. DIJIN'S TRAVELS

Dijin is late to his calculus class again! However, he is in no hurry. Below is a graph of the *velocity* (in meters per minute) Dijin travels during the first 4 minutes of his journey.



a. Estimate Dijin's velocity at $t = 3.5$ minutes.

b. If his velocity is measured in meters per minute, while the time is measured in minutes, what are the appropriate units for the area under the curve? Why?

c. What does the area under a velocity curve represent.

d. How far did Dijin travel during the first 4 minutes?

e. What was Dijin's average velocity during the first 4 minutes?

f. Sketch Dijin's graph on your paper. On the same set of axes, sketch the line $y = avg\ velocity$ (from part (e)) and shade the area under the curve. What shape is this?

g. How many times did Dijin travel at his average velocity. Justify your answer.

h. If Dijin's initial position was 100 meters away from the classroom and he continues to travel at the same rate he was traveling at $t = 4$, sketch a graph of his velocity. How many minutes would it take him to get to class?

1-163. Due to eye-fatigue, a person's reading rate decreases after 2 hours. The rate (in words per hour) of a certain reader is represented by the piecewise function below.

$$\text{reading rate} = \begin{cases} 6000 & \text{for} \quad 0 \le t \le 2 \text{ hrs} \\ 7000 - 500t & \text{for} \quad t > 2 \text{ hrs} \end{cases}$$

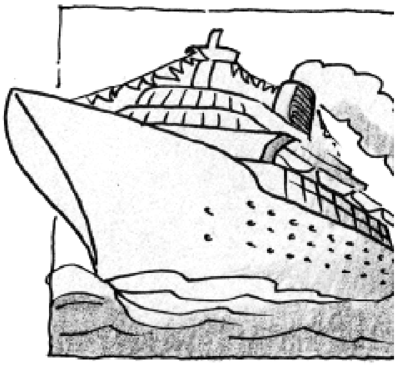
a. Sketch a graph of this reading rate model.

b. According to this model, how many words can this person read in 5 hours? How did you find your answer?

c. What is the average number of words per hour a person reads over this five-hour interval?

1-164. Summarize your method for finding how much something has changed given its rate of change. For example, given a rate such as how fast a person types, how would you be able to find the number of words typed? Explain why your method works.

1-165. TRIP TO BAJA



Ms. D went on an awesome cruise to Baja, California this summer. One morning, at 8:00 a.m., she was informed that the cruise ship was traveling at a rate of 39 miles per hour.

She created the table below and recorded the speed of the ship at 15-minute intervals.

a. Draw a speed-time graph and estimate the distance traveled between 8 a.m. and 10 a.m. Make your estimate as accurate as possible.

Time (hrs)	8:00	8:15	8:30	8:45	9:00	9:15	9:30	9:45
Speed (mph)	39	34	26	23	15	18	26	27

b. If an Asia-bound cruise left from Catalina Island, which is 22 miles from Los Angeles Harbor, approximately how far from LA was the ship at 10 a.m.?

1-166. Theo left his motion graph at home. Fortunately, while he had the graph, he determined the properties shown below. Help him re-create a possible distance-time graph of his motion.

DETAILS:

- Theo walked in one direction during the entire experiment.
- His average velocity during the first half of the time was 5 feet per second. His average velocity for the second half was only 3 feet per second.
- He started while standing 2 feet from the motion detector and began to walk away from it at $t = 0$. He stopped when he was 26 feet from the motion detector.

Review & Preview

1-167. Oil is leaking out of a car at a rate of $y = 0.2^t$ liters/hour for $0 \le x \le t$.

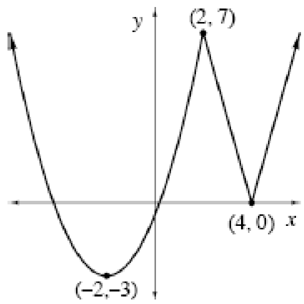
a. Sketch a graph of this situation. Label the units on the axes.

b. Shade the area under the curve for $0 \le t \le 1$. What does the shaded area represent? What are the units?

1-168. Find the equation of the line through the point (3, −2) with a slope of 7. Leave your answer in point-slope form.

1-169. Revisit your rates of change from problem 1-143. Decide what measurement would be determined if the area under graphs of these rates were calculated.

1-170. Use a parabola and absolute value function to find a piecewise defined function that will produce the graph shown at right:



1-171. Write a complete set of approach statements for $y = \frac{(2x+1)(2-x)}{2x+1}$.

1-172. A flag is defined by the region between the x-axis and the function listed below. Sketch the flag and find the volume when the flag is rotated about the x-axis.

$$f(x)=\begin{cases} 2 & \text{for } 0 \le x \le 2 \\ 4-x & \text{for } 2 < x \le 4 \end{cases}.$$

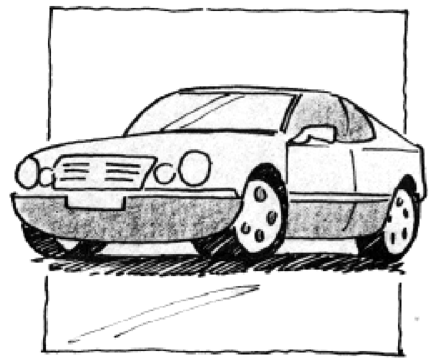
1-173. For $f(x)=\frac{x^2-2x-3}{x-3}$ and $g(x)=-x+2$, find and simplify $h(x)=f(g(x))$.

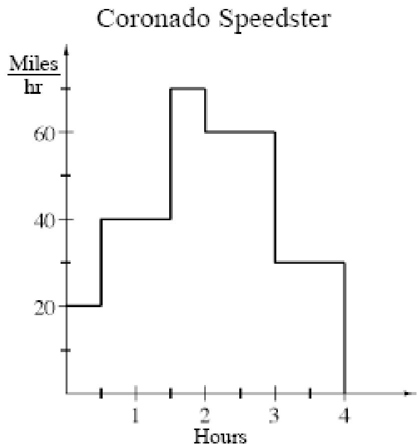
a. Graph $h(x)$ and write its domain in set notation.

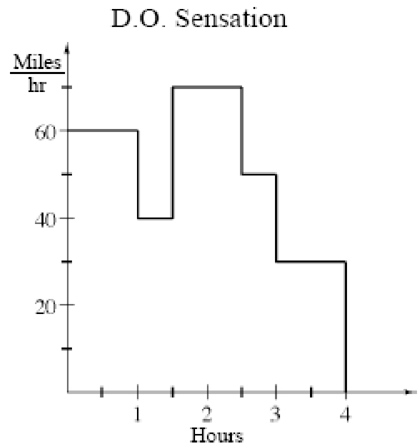
b. Find $b(x)$, the end behavior function of $h(x)$.

1-174. TEST DRIVE

Sarah wants to buy a new car and is deciding between two models. She has convinced both car dealerships to allow her to test drive each car for 4 hours as long as she returns with a full tank of gas. In order to test the performance of both vehicles, she kept track of her velocity during her test drive every half hour. The results of her test drives are shown below.

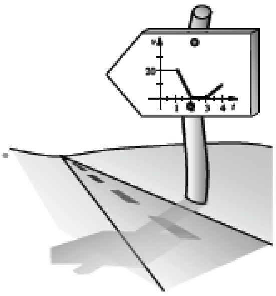






Sarah finds that both cars performed exceptionally well and she would be very happy with either one. She decided to make her final decision based on the gas mileage of each model. Her test drive of the Coronado Speedster used 7.955 gallons of gas and her test drive of the D.O. Sensation used 8.542 gallons. Which car should she choose?

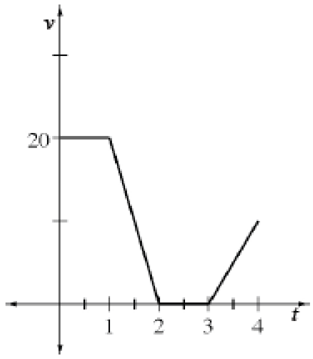
1-175. A focus of this course will be determining maximum and minimum values for a function on a given interval. However, you already have the skills to do this for certain functions. On your paper, sketch $y=2\sin(3x)$ for $0 \le x \le \frac{2\pi}{3}$. Find the points at which y is a maximum and minimum.



1.4.4 How does velocity change?

Acceleration

1-176. Problems 1-162 and 1-163 each started with a **rate of change**. For example, Dijin's velocity describes how his position changes, while the rate in problem 1-163 focused on how the number of words being read changes. Now consider how these *rates of change* change themselves!



a. Dijin's velocity in meters/min is shown again in the graph at right. Carefully describe how his velocity changes.

b. The rate at which the velocity changes at is called **acceleration**. Of Dijin's velocity is measured in meters per minute, what would the units of Dijin's acceleration be? Explain.

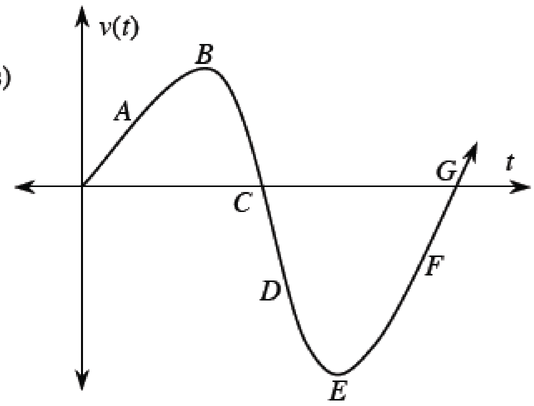
c. Write a piecewise function to represent Dijin's acceleration, $a(t)$, for $0 \le t \le 4$.

d. What happens to Dijin's velocity when his acceleration is zero?

e. During what interval was Dijin's acceleration positive? How do you know? Describe Dijin's motion during this interval.

f. Describe what acceleration will tell you about Dijin's motion.

1-177. Use the information from the velocity graph at right, to decide at which point(s) the given events happen.



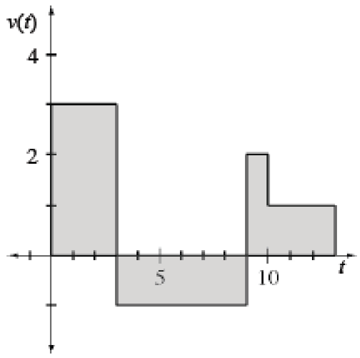
a. Velocity is positive.

b. Speed is greatest. (Note—it may help to sketch a speed graph.)

c. Acceleration is negative.

d. Velocity is zero.

1-178. Betsy rode her bike and generated the velocity graph at right. Assume $v(t)$ is measured in miles per hour and t is measured in hours.

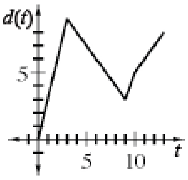


a. Find the area under $v(t)$. What does this area represent?

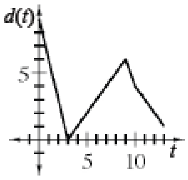
b. The distance from part (a) represents Betsy's displacement, not her total distance. Sketch another graph that represents her total distance.

c. Which of the graphs below could represent Betsy's distance from home? There may be more than one answer.

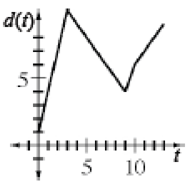
(A)



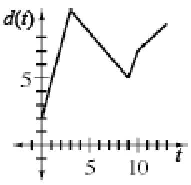
(B)



(C)

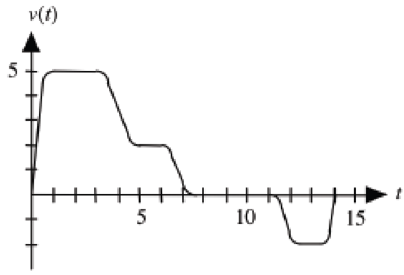


(D)



d. If Betsy's initial position was 2 miles from her home, which graph represents her position as a function of time? How far from home was she at $t = 13$?

1-179. The graph at right shows the velocity of a bug crawling back and forth along the x-axis. Assume that velocity is measured in feet per minute.



a. Describe the motion of the bug.

b. What happens to the bug's motion when the velocity is negative?

c. Approximately how far did the bug travel in the first 7 minutes?

d. What is the bug's displacement?

e. What is the total distance traveled by the bug during the first 14 minutes?

f. Sometimes the displacement equals the total distance. Under what conditions are these measurements the same?

g. Jessica, Yoo, and Carl are afraid of bugs. They want to know the bug's position on the x-axis at $t = 14$. Jessica thinks the bug is at (20, 0). Yoo says the bug is at (28, 0). Carl says that there is not enough information to answer the question. Who is right? Explain.

h. If the bug's initial position at $t = 0$ was (–7,0), where is the bug at $t = 14$?

1-180. While Bungee jumping, Rajeesh noticed different times during which his motion met the following conditions. For each condition, describe the motion that was occurring. Assume displacement is measured as his height above the ground.

a. Positive velocity with negative acceleration.

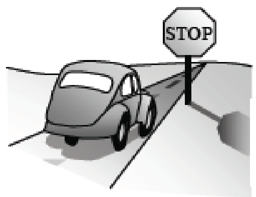
b. Negative velocity with positive acceleration.

c. Positive velocity with no acceleration.

d. Zero velocity with a negative acceleration.

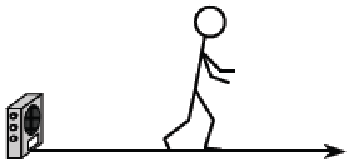
e. At one point Rajeesh was falling while slowing down. Is the velocity positive or negative? What about the acceleration?

Math Notes



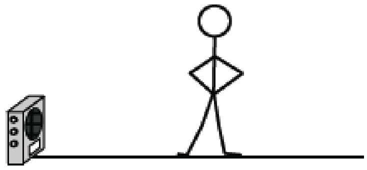
The Vocabulary of Motion

As an object moves, its position with respect to its starting place changes. The rate of this change is the **velocity** of the object. If the distance is increasing, the object is moving away from the starting place and the velocity is positive. Likewise, if the distance is decreasing, the object is moving toward the starting place and the velocity is negative. When motion is stopped, the velocity is zero. If the motion is vertical, traditionally up is positive.



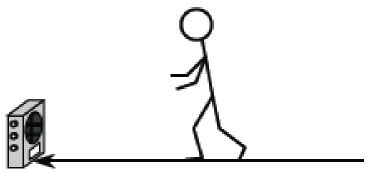
velocity > 0

When the direction of the motion is not an issue, the rate of motion can be called **speed**. Speed is always positive and is the absolute value of the velocity.



velocity $= 0$

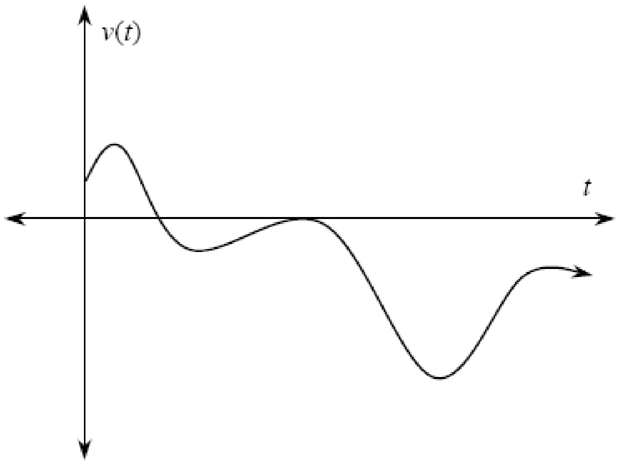
The **acceleration** is the rate of change of the velocity of the object. The acceleration measures the rate at which something is "speeding up" or "slowing down." If acceleration is zero, then the velocity is constant.



velocity < 0

When the **displacement** is divided by **total time**, the result is the **average velocity**. Similarly, when the **total distance** is divided by **total time**, the result is the **average speed**.

1-181. The graph below represents velocity of an object as a function of time. Trace it on your paper.



a. Put a star * at the point where velocity is the greatest.

b. With another color, sketch a graph of the speed on the same set of axes.

c. Indicate with a double star ** the position where speed is the greatest.

d. Explain why the greatest velocity and the greatest speed do not occur at the same position.

e. Sketch a new graph where speed and velocity have the same maximum value.

1-182. Marni loves pancakes and likes to eat them in a tall stack. Assume that she always makes 8 pancakes with a thickness of $\frac{1}{4}$ inch.

a. Last Saturday, Marni decided to make square pancakes. If the largest pancake had an edge of 9 inches and each pancake had an edge $\frac{1}{4}$ inch shorter than the one below it, find the volume of pancakes Marni ate last Saturday.

b. Next Sunday, Marni will make circular pancakes. This time, each pancake will have a diameter 1 inch smaller than the one below it and the smallest pancake will have a radius of 2 inches. How much volume is she planning to eat?

1-183. Theo lost his graph again! Luckily, he used his distance-time graph to determine the following properties of his motion. Help him re-create a possible graph of his motion.

DETAILS:

- He walked in one direction during the entire 5 seconds, except during the 2 seconds when he was temporarily still.
- His average velocity was −2 feet per second.
- He began his motion 12 feet away from the motion detector.

1-184. As a cheetah runs, its velocity is $v(t)=\begin{cases} 12t^2 & \text{for } 0\le t\le 2 \\ -t^2+52t-52 & \text{for } t>2 \end{cases}$ where $v(t)$ is measured in feet per second.

a. Sketch a graph of $v(t)$ on your paper.

b. Approximately how fast is the cheetah running at $t = 1$ second? How did you get your answer?

c. To catch prey, such as an antelope, the cheetah runs for 3 seconds. Approximately how far does it need to run to catch its prey? Describe your method.

d. If the cheetah spots Annalou the Antelope standing 50 feet north of the Great Pond and runs north in a direct line towards her, catching her in exactly 3 seconds, what was the cheetah's initial position relative to the pond?

1-185. Rewrite $f(x)=|x^2-4|+1$ as a piecewise function. State the domain and range of $f(x)$ using interval notation.

1-186. Find $b(x)$, the end behavior function of $f(x)=\frac{x^3+3x^2-4x-1}{x^2-1}$. Then, write a complete set of approach statements for $f(x)$.

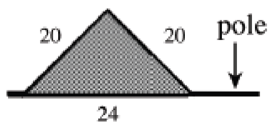
1-187. Solve the following for all values of x in the domain $(0, 2\pi)$. Use exact values.

a. $\sin(2x) = \sin(x)$

b. $\sin(x+\pi) + \cos\left(x - \frac{\pi}{2}\right) = 1$

c. $\cot x - \tan x = 2\sqrt{3}$

1-188. The shaded region at right represents a triangular "flag."



a. Imagine rotating this flag about its "pole" and describe the resulting three-dimensional figure. Draw a picture of this figure on your paper.

b. Find the volume of the rotated flag.

1-189. Let $g(x) = (3x-1)^2$.

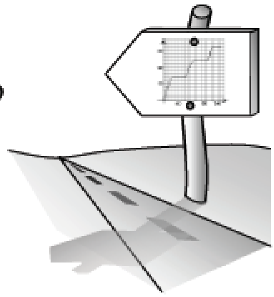
a. Find $g(-5)$.

b. Find $g(a+1)$ and simplify.

c. If $g(x) = 49$, find x.

d. Find $g^{-1}(x)$.

1.5.1 How do position and velocity relate?

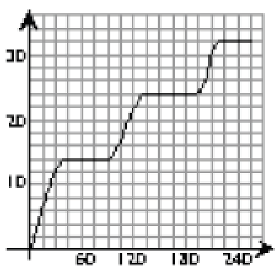


Area and Slope

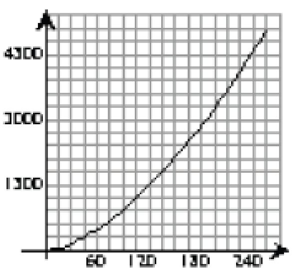
1-190. FREDO AND FRIEDA RETURN

Fredo and Frieda each recorded data for a different race. Their data is shown below and this time the students are not so sure their data matches. The coach has turned to you for help. Decide which student measured distance and which measured velocity. Create ways to decide if the graphs represent the same data. Be prepared to share your methods to confirm whether the data collected by each student matches the others.

Fredo's Graph



Frieda's Graph



1-191. College Admissions Writing Prompt

Liebniz University in Newton, North Calculina has a unique requirement for math majors. In order to receive credit for scoring well on the AP Calculus exam, students must demonstrate understanding of the following topic through a formal essay.

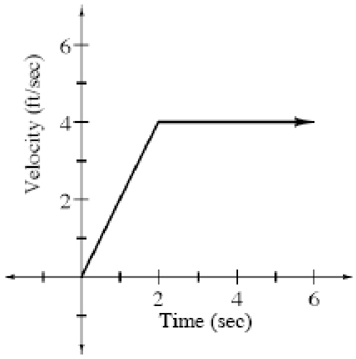
Please respond to the following prompt. Note: If you include graphs to illustrate your ideas, make sure they are well labeled.

What is the relationship between position and velocity?

- How can velocity information be determined from a position graph? What about speed?
- How can position information be determined from a velocity graph?

Be sure to draw distinctions between actual position, displacement, and total distance. What extra information do you need to determine actual position from a velocity graph?

1-192. So far in this chapter, you have determined information about acceleration and distance from a velocity graph. You have developed ways to find total distance or displacement. This time, however, we want to find a function of the distance over time. Using the graph:



a. Find a piecewise function for $v(t)$.

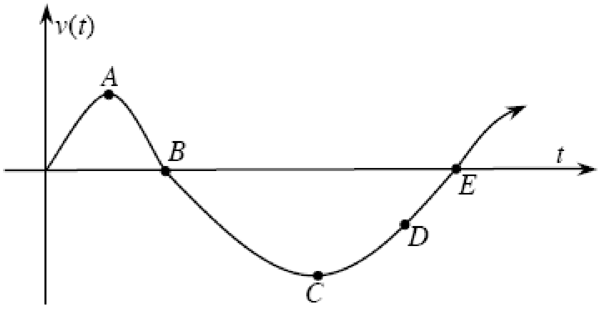
b. Assuming that $d(0) = 0$, copy and complete the table below for distance traveled over time.

t	0	0.5	1	1.5	2	3	4	5	6
d(t)					4				20

c. Plot the distance verses time graph $d(t)$ from the table completed in part (b).

d. Assuming that $d(0) = 0$, find the piecewise function for the distance versus time graph.

1-193. A particle is moving along a straight line. The velocity of the particle is shown on the graph below.



a. At what point is the velocity greatest?

b. At what point is the speed greatest?

c. Where does the particle change from moving forward to moving backward?

d. Where is acceleration positive?

Review & Preview

1-194. NOT AGAIN!

Theo has done it again. Before he lost his graph, he used it to determine the following properties of his motion. Help him re-create a possible graph of this motion.

DETAILS:

- He changed directions three times during his 8-foot walk.
- His average velocity was 0 feet per second.
- Theo walked for 6 seconds and started 5 feet from the motion detector.

1-195. Carefully graph $f(x)=\begin{cases}\sqrt{x} & \text{for } 0<x<4\\ (x-6)^2-2 & \text{for } 4\le x<10\end{cases}$ on your paper. Then, write a detailed slope statement.

1-196. Given the tables below,

x	-2	-1	0	1	2	3	10	100
$f(x)$	-11	-8	-5	-2	1	4	25	295

x	-3	-2	-1	0	1	2	3	12
$g(x)$	-5	0	3	4	3	0	-5	-140

x	-2π	$-\pi$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	12π
$h(x)$	2	-2	2	0	-2	0	2	2

a. Find possible functions for $f(x)$, $g(x)$, and $h(x)$.

b. Evaluate:

i. $f(g(h(\pi)))$ *ii.* $h(g^{-1}(4))$ *iii.* $f^{-1}(h(\pi))$

1-197. The population of Smalltown on January 1st for 5 years is shown in the table at right.

1995	2300
1996	2415
1997	2536
1998	2663
1999	2796

a. Write a slope statement for the given data.

b. Find the average rate of growth of the population between 1995 and 1999.

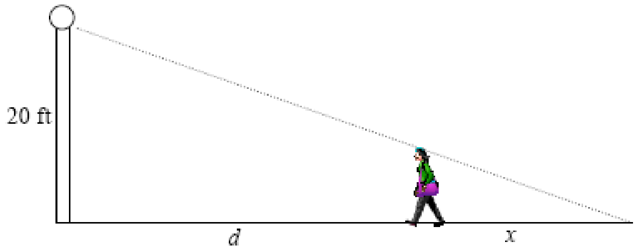
c. Approximate the rate of increase of the population on January 1st, 1998. Explain how you got your answer.

1-198. Draw *two different* distance graphs that each has an average velocity of 5 meters per minute.

1-199. The between the x-axis and $f(x) = -|x-3|+5$ forms a flag. Find the volume generated when the flag is rotated about the x-axis.

1-200. Find non-trivial functions $f(x)$ and $g(x)$ such that $f(g^{-1}(x)) = \sqrt{3x-2}+6$.

1-201. Shehazana, who is five and a half feet tall, is walking toward a 20-foot streetlight. Solve for the length of her shadow, x, in terms of the distance, d, she is from the pole.



1-202. Find the domain of the following functions:

a. $f(x) = \sec x$

b. $g(x) = \log(x^2 + 1)$

c. $h(x) = \frac{x^2-4}{x^2-x-6}$

d. $k(x) = \frac{\log(x-1)}{\sqrt{x^2-16}}$

1-203. **Multiple Choice:** When the graph of $f(x) = 1 - 2^x$ is reflected across the y-axis, the resulting graph is:

a. $g(x) = 1 - 0.5^x$

b. $g(x) = 1 + 2^x$

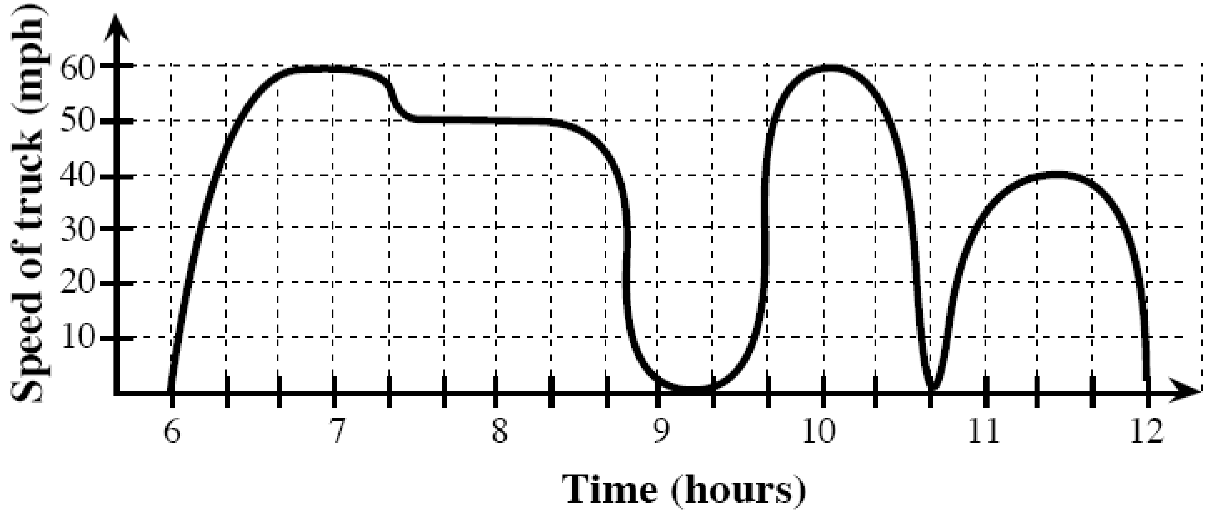
c. $g(x) = 2^x - 1$

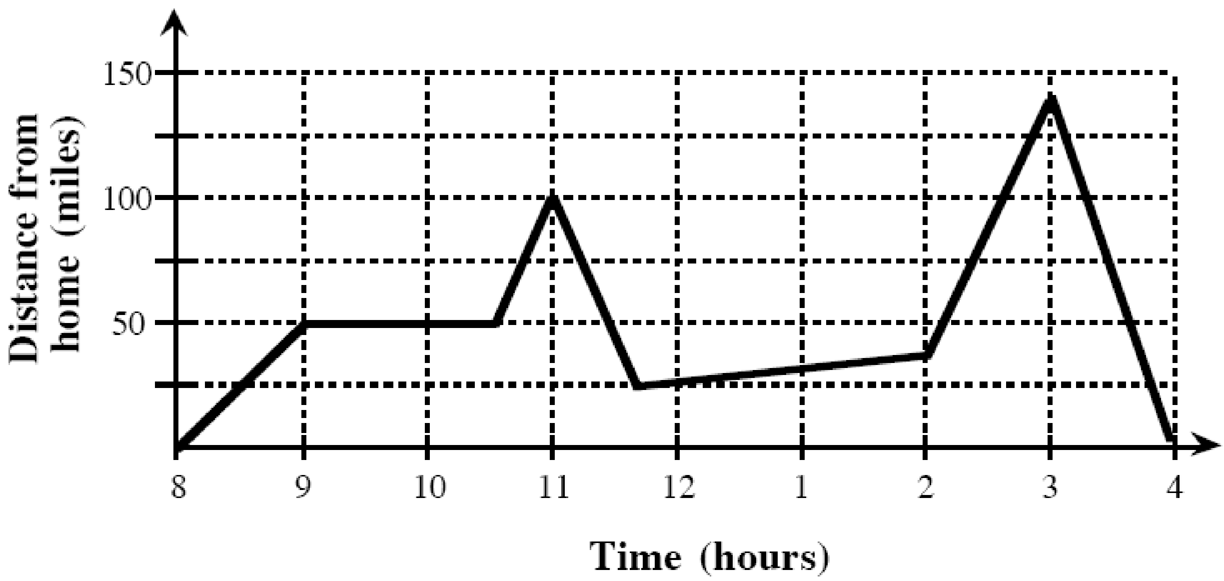
d. $g(x) = \log_2(x-1)$

e. $g(x) = \log_2(1-x)$

Lesson 1.1.1 Resource Page

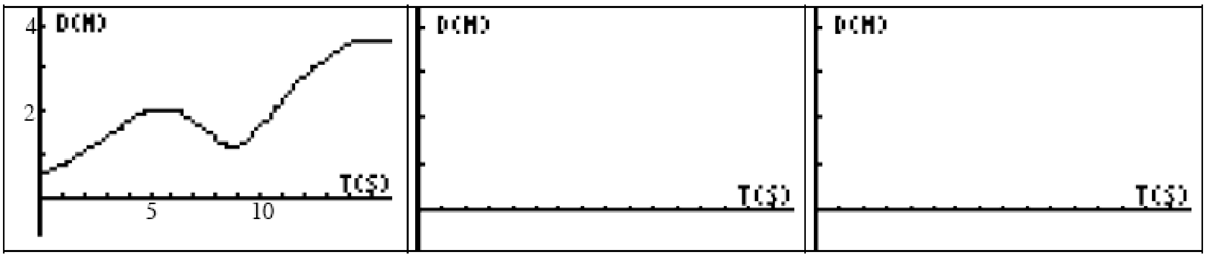
Freeway Fatalities





Lesson 1.4.2 Resource Page

WEARY VERONICA



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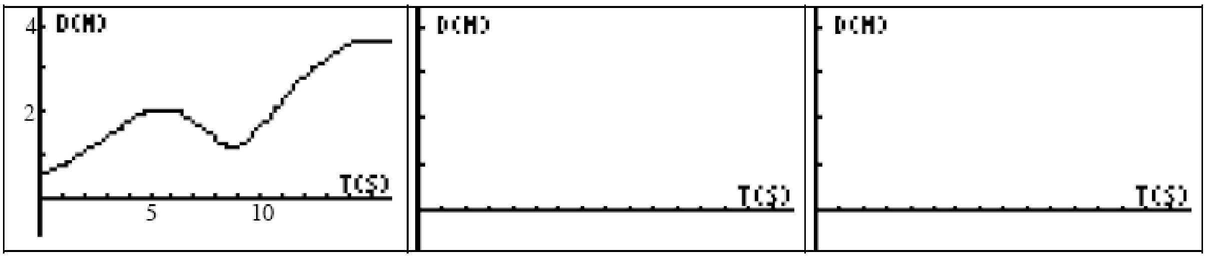

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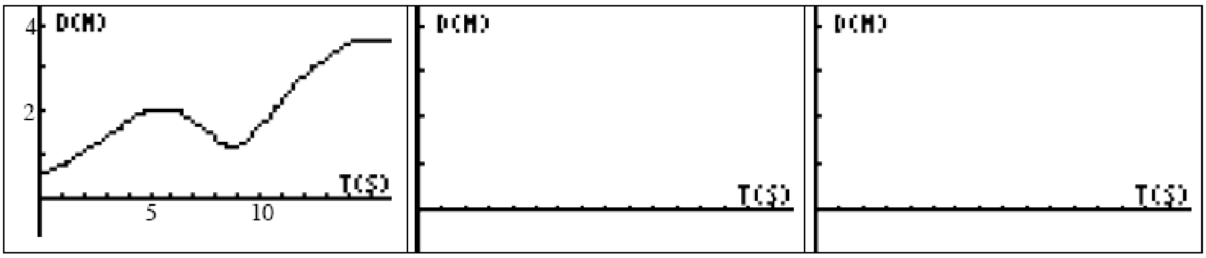

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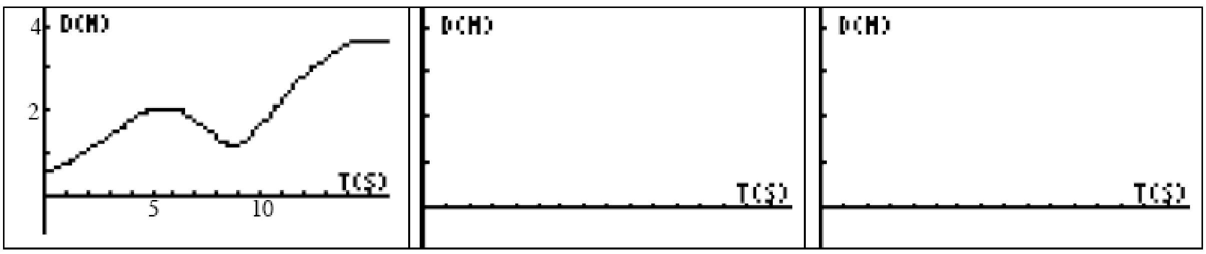
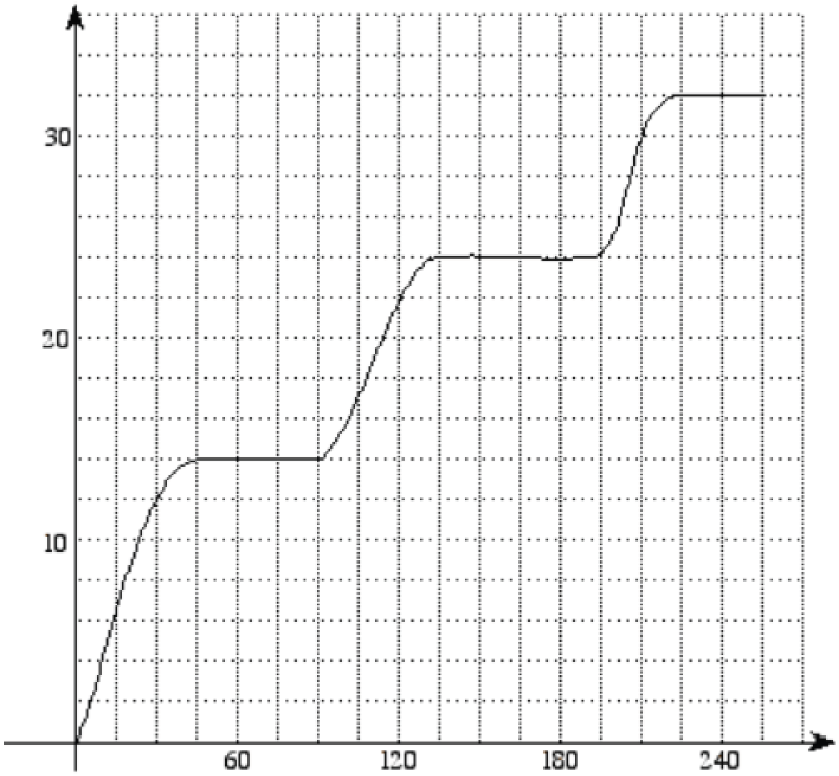


Lesson 1.5.1 Resource Page

FREDO'S GRAPH



FRIEDA'S GRAPH